VII. Probability (Part IV: Continuous Probability Distributions)

- Normal Distribution
- Other Continuous Distributions
- Overview: Discrete vs. Continuous

Normal Distribution (1/6)

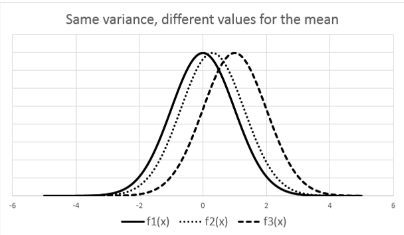
- The total area under the curve is one ($\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x = 1$). The function $f(x)$ is called the probability density function.

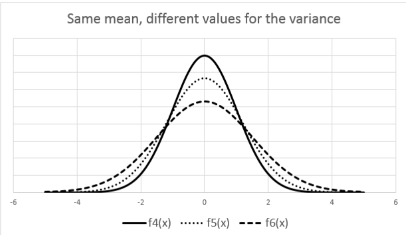
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{1}{2}\left(\frac{(x-\mu)}{\sigma}\right)^2}$$

- The curve is symmetric around μ (The area on each side of the mean equals 0.5 or 50%).
- The tails of the curve extend indefinetely
- If a random variable X follows a normal distribution with μ and σ, we write $X \sim N(\mu, \sigma)$. The parameters μ and σ determine the shape of the curve. The mean μ is responsible for the location and σ ($\sigma > 0$) the shape. The larger σ, the flatter and wider the curve.
- The highest point of $f(x)$ occurs at the mean value μ
- Most important distribution for continuous random variables
- We can determine the probability that a value is within a given interval [a, b]
- Probabilities can only be determined approximately (with tables, Software)

Normal Distribution (2/6) - Effects of μ and σ

Remember the notation $X \sim (\mu, \sigma)$





- $f_1(x) \ldots X \sim N(0, 1)$
- $f_2(x) \ldots X \sim N(0.3, 1)$
- $f_3(x) \ldots X \sim N(1, 1)$
- $f_4(x) \ldots X \sim N(0, 1)$
- $f_5(x) \ldots X \sim N(0, 1.2)$
- $f_6(x) \ldots X \sim N(0, 1.5)$

Normal Distribution (3/6): Using Excel

EXCEL-Functions

=NORM.DIST(x;μ;σ;1)	Gives **the probability P(X < x)** if $X \sim N(\mu, \sigma)$
=NORM.INV(prb;μ;σ)	Gives the **the value x** so that P(X < x) = prb

Example: Using NORM.DIST

Problem

The results of a national survey showed that on average, adults sleep 6.9 hours per night. Suppose that the standard deviation is 1 hour. Calculate the percentage of individuals who sleep ...

- more than 8 hours.
- less than 5 hours.
- between 6 and 7 hours.

Solution

- =1 - NORM.DIST(8;6.9;1;1) = 13.57%
- =NORM.DIST(5;6.9;1;1) = 2.87%
- =NORM.DIST(7;6,9;1;1)-NORM.DIST(6;6,9;1;1) = 35.58%

Normal Distribution (4/6): Using Tables

If a random variable X follows a normal distribution, $X \sim N(\mu, \sigma)$

The transformation

$$Z = \frac{x - \mu}{\sigma}$$

leads to a **standard normal distribution**.
Now,

$$Z \sim N(0, 1)$$

Tables for a **standard normal distribution** can be found in any textbook about introductory statistics or online as well as in formularies.

Normal Distribution (5/6): Examples

Example: Using NORM.INV

Problem
The IQ is normally distributed with mean of 100 and standard deviation of 15.

1. What is the critical value that cuts off the lowest 10% of all IQ scores?
2. What is the critical value to belong to the best 1% of all IQ scores?
3. What is the critical value to belong to the best 25% of all IQ scores?

Round the values to the next integer.
Solution

1. =NORM.INV(0,1; 100;15) = ≈ 81 IQ-Points
2. =NORM.INV(0,99; 100;15) = ≈ 135 IQ-Points
3. =NORM.INV(0,75; 100;15) = ≈ 110 IQ-Points

Normal Distribution (6/6): Examples (continued)

Example 1

Problem Let X be a cont. random variable, $X \sim N(0, 1)$. Find $P(x < 0.2)$

Solution 57.93%

Example 2

Problem Let X be a cont. random variable, $X \sim N(100, 15)$.

1. Find the probability that $x > 110$
2. Find the probability that $x < 95$

Solution 1. ≈ 25.25%, 2. ≈ 36.94%

Example 3

Problem The amount that airlines spend on food per passenger is normally distributed with $\mu = 4{,}50$ EUR and $\sigma = 0{,}80$ EUR. What percent spend more than EUR 6,00?

Solution 3.04%

Other Continuous Distributions (1/1)

In the list below you can find some continuous distributions. We will come accross those in bold print in our last lessons:

- **χ^2 distribution**
- F-distribution
- **t-distribution**
- Uniform distribution
- ...

Overview: Discrete vs. Continuous (1/1)

Discrete

- Finite number of possible outcomes
- Assigns a probability to each possible outcome
- To find the probability that a value between a and b occurs, we can add the respective probabilities.

Continuous

- Infinite number of possible outcomes
- To find the probability that a value between a and b occurs, we can calculate the area under the curve of the corresponding density function.
- We can obtain the area under a density function f(x) between the values a and b by by calculating $\int_a^b f(x)$ dx.
- Usually those areas (=probabilities) can be found approximately only (using tables or software).