Exercise Sheet 8: Integral in One Dimension

Example 1 *Integrate the function* $f(x) = x^2$ *in the interval* $[1, 2]$.

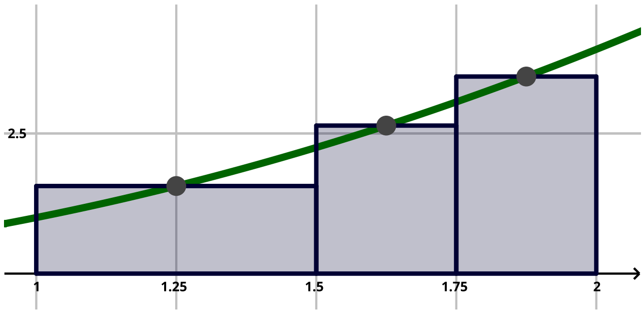

Figure 1: Midpoint rule

a) *Compute the exact value.*

b) *Midpoint rule: Use the three intervals (see figure 1) and approximate the function in each interval by the function value of the corresponding midpoint.*

c) *Compute the relative error of the midpoint rule.*

Solution: $A = 2.\dot{3}$, $A_{mid} \approx 2.32$, $rel \approx 0.56\%$

Example 2 *Compute the highlighted area of figure 2.*

Solution: $A = 101,25$

Example 3 *Compute the highlighted area of figure 3. Attention: Both parts have to be positive.*

Solution: $A = 8$

Example 4 *Solve the integrals*

$$I_1 = \int 2x \cdot e^{3x^2}\,\mathrm{d}x, \qquad I_2 = \int x \cdot e^x\,\mathrm{d}x, \qquad I_3 = \int \frac{2x+3}{\sqrt{x^2+3x-1}}\,\mathrm{d}x,$$

$$I_4 = \int 4 \cdot \sin(\cos(x)) \cdot \sin(x)\,\mathrm{d}x, \quad I_5 = \int 5x \cdot \cos(x)\,\mathrm{d}x, \quad I_6 = \int x^2 \cdot e^x\,\mathrm{d}x$$

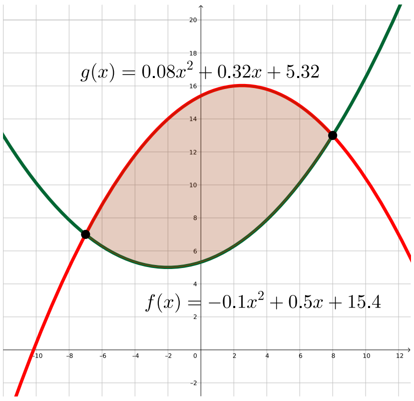

Figure 2: Fläche

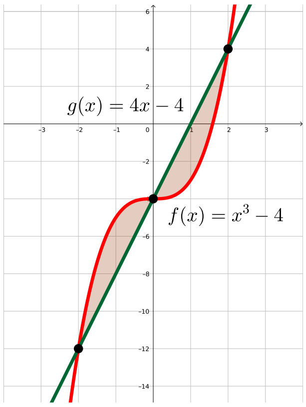

Figure 3: Fläche

by using an appropriate method. Check your results with Python or compute the derivative of your result to check its correctness.

Finally, compute the integrals:

$$A_1 = \int_0^1 2x \cdot e^{3x^2} \, \mathrm{d}x, \qquad A_2 = \int_0^1 x \cdot e^x \, \mathrm{d}x, \qquad A_3 = \int_1^2 \frac{2x+3}{\sqrt{x^2+3x-1}} \, \mathrm{d}x,$$

$$A_4 = \int_0^1 4 \cdot \sin(\cos(x)) \cdot \sin(x) \, \mathrm{d}x, \qquad A_5 = \int_0^1 5x \cdot \cos(x) \, \mathrm{d}x, \qquad A_6 = \int_0^1 x^2 \cdot e^x \, \mathrm{d}x.$$

Solution:
$I_1 = \frac{e^{3x^2}}{3} + c$ *and* $A_1 \approx 6.36$
$I_2 = (x-1) \cdot e^x + c$ *and* $A_2 = 1$
$I_3 = 2\sqrt{x^2+3x-1} + c$ *and* $A_3 \approx 2,54$
$I_4 = 4\cos(\cos(x)) + c$ *and* $A_4 \approx 1,27$
$I_5 = 5x \cdot \sin(x) + 5\cos(x) + c$ *and* $A_5 \approx 1,91$
$I_6 = \left(x^2 - 2x + 2\right) \cdot e^x + c$ *and* $A_6 \approx 0,72$