Newton's Method and Taylor Polynomials

Exercise 1 *Write the formula for Newton's method and use the given initial approximation to compute the approximations x_1 and x_2.*

a) $f(x) = x^2 - 2x - 3; x_0 = 2$

b) $f(x) = e^{-x} - x; x_0 = \ln 2$

Exercise 2 *Use a calculator or program to compute the first 10 iterations of Newton's method when it is applied to the following functions with the given initial approximation. Make also a table with the values of n and x_n.*

a) $f(x) = x^3 + x^2 + 1; x_0 = -2$

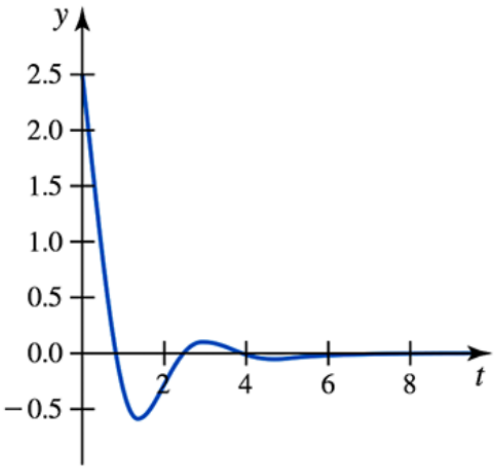
b) $f(x) = \sin x + x - 1; x_0 = 1.5$

c) $f(x) = \ln(x + 1) - 1; x_0 = 1.7$

Exercise 3 *Where is the local extremum of* $f(x) = \dfrac{e^x}{x}$ *located? (Use Newton's method.)*

Exercise 4 *The functions* $f(x) = (x-1)^2$ *and* $g(x) = x^2 - 1$ *both have a root at* $x = 1$. *Apply Newton's method to both functions with an initial approximation* $x_0 = 2$. *Compare the rate at which the method converges in each case and give an explanation.*

Exercise 5 *The displacement of a particular object as it bounces vertically up and down on a spring is given by* $y(t) = 2.5e^{-t}\cos 2t$, *where the initial displacement is* $y(0) = 2.5$ *and* $y = 0$ *corresponds to the rest position (see figure).*



a) Find the time at which the object first passes the rest position $y = 0$.

b) Find the time and the displacement when the object reaches its lowest point.

c) Find the time at which the object passes the rest position for the second time.

d) Find the time and the displacement when the object reaches its high point for the second time.

Exercise 6 *Find the n-th order Taylor polynomials of the given function centered at 0, for $n = 0, 1,$ and 2. Graph the Taylor polynomials and the function.*

a) $f(x) = \cos x$

b) $f(x) = \ln(1 - x)$

c) $f(x) = (1 + x)^{-3}$

Exercise 7

1. *Find the Taylor polynomials of order $n = 0, 1, 2$ and 3 for $f(x) = e^x$ centered at 0. Graph f and the polynomials.*
2. *Use the polynomials in part (a) to approximate $e^{0.1}$. Find the absolute error, $|f(x) - p_n(x)|$, in the approximation. Use calculator values for the exact values of f.*

Exercise 8 *Approximate $\sqrt{1.05}$ using $f(x) = \sqrt{1 + x}$ and $p_2(x) = 1 + \frac{x}{2} - \frac{x^2}{8}$.*

Compute the absolute error in the approximation assuming the exact value is given by a calculator.

Exercise 9 *Find the n-th order Taylor polynomials for the following functions centered at the given point a, for $n = 0, 1,$ and 2. Graph the Taylor polynomials and the function.*

a) $f(x) = x^3, a = 1$

b) $f(x) = \sin x, a = \frac{\pi}{4}$

c) $f(x) = \sqrt{x}, a = 9$

d) $f(x) = \ln x, a = e$

e) $f(x) = \tan^{-1} x + x^2 + 1, a = 1$

Exercise 10 *Use the power series expansion of e^x to show that*

$$e^{2x} = 1 + 2x + 2x^2 + \frac{4x^3}{3} + ...$$