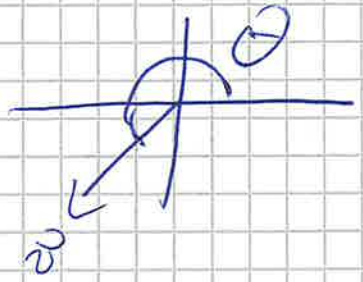
Sheet 3 Exercise 11

$f(x,y) = 10 - 2x^2 - 3y^2$ $\quad P = (3,2)$

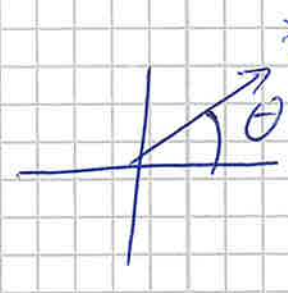
a) $\nabla f(x,y) = \begin{pmatrix} -4x \\ -6y \end{pmatrix}$ $\quad \nabla f(3,2) = \begin{pmatrix} -12 \\ -12 \end{pmatrix}$

b) max: $\vec{v} = \begin{pmatrix} -12 \\ -12 \end{pmatrix}$



$\theta = 225^\circ$

min: $\vec{v} = -\begin{pmatrix} -12 \\ -12 \end{pmatrix} = \begin{pmatrix} 12 \\ 12 \end{pmatrix}$



$\theta = 45^\circ$

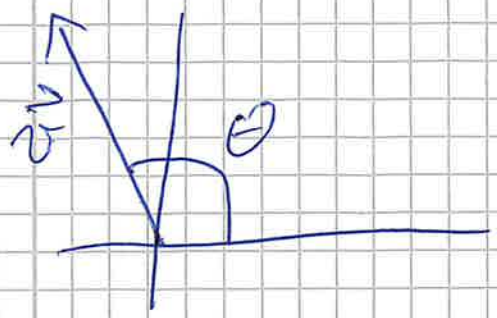
no change: $D_{\vec{v}}(3,2) = \begin{pmatrix} -12 \\ -12 \end{pmatrix} \cdot \frac{\vec{v}}{|\vec{v}|}$

$\vec{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\quad \theta = 315^\circ$

or

$\vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $\quad \theta = 135^\circ$

c) $D_{\vec{v}}(B,2) = \begin{pmatrix} -12 \\ -12 \end{pmatrix} \cdot \frac{\vec{v}}{|\vec{v}|} = -12\cos\theta - 12\sin\theta$



$$\frac{\vec{v}}{|\vec{v}|} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

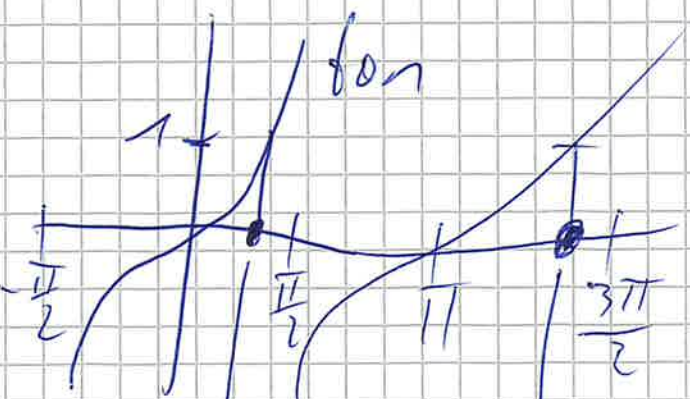
$$g(\theta) = -12\cos(\theta) - 12\sin(\theta)$$

d) $g'(\theta) = 12\sin(\theta) - 12\cos(\theta) \overset{!}{=} 0$

$$12\sin(\theta) = 12\cos(\theta)$$

$$1 = \frac{\cos(\theta)}{\sin\theta} = \tan(\theta)$$

$$\theta_1 = \arctan(1) = 0.785\ldots \text{ rad}$$



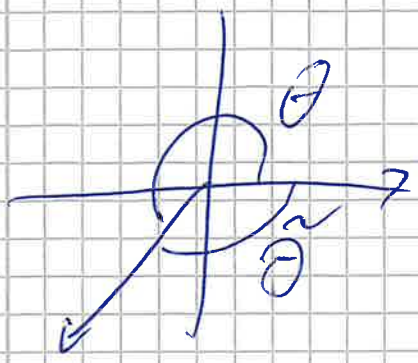
$g(\theta_1) = -16,970\ldots$ ↑ Min!

0,785...

3,926...

$\theta_2 = \theta_1 + \pi = 3,926\ldots$ $g(\theta_2) = 16,970\ldots$ ↓ Max!

c) $\nabla f(3,2) = \begin{pmatrix} -12 \\ -12 \end{pmatrix}$



$$\cos(\tilde{\theta}) = \frac{\left|\begin{pmatrix} -12 \\ -12 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right|}{\left\|\begin{pmatrix} -12 \\ -12 \end{pmatrix}\right\| \cdot \left\|\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\|} = \frac{-12}{\sqrt{2} \cdot 12}$$

$$\tilde{\theta} = \arccos\left(\frac{-1}{\sqrt{2}}\right) = 2,356\ldots \text{ rad}$$

$$\theta = 2\pi - \tilde{\theta}$$
$$= 3,926\ldots \text{ rad}$$
$$= 225^\circ$$

(cf b)

$$\left|\nabla f(3,2)\right| = \left\|\begin{pmatrix} -12 \\ -12 \end{pmatrix}\right\| = \sqrt{2} \cdot 12$$
$$= 16,970\ldots$$

$$f(3,926\ldots) = -12\cos(\theta) - 12\sin(\theta)$$
$$= 16,970\ldots$$

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