Tangent Plane

Exercise 1 *Find an equation of the plane tangent to the following surfaces at the given points (two planes and two equations).*

a) $x^2 + y + z = 3$; $(1, 1, 1)$ *and* $(2, 0, -1)$

b) $xy + xz + yz - 12 = 0$; $(2, 2, 2)$ *and* $(2, 0, 6)$

c) $xy \sin z = 1$; $(1, 2, \pi/6)$ *and* $(-2, -1, 5\pi/6)$

d) $yze^{xz} - 8 = 0$; $(0, 2, 4)$ *and* $(0, -8, -1)$

e) $z^2 - \dfrac{x^2}{16} - \dfrac{y^2}{9} - 1 = 0$; $(4, 3, -\sqrt{3})$ *and* $(-8, 9, \sqrt{14})$

f) $z = 4 - 2x^2 - y^2$; $(2, 2, -8)$ *and* $(-1, -1, 1)$

g) $z = 2 + 2x^2 + \dfrac{y^2}{2}$; $(-\dfrac{1}{2}, 1, 3)$ *and* $(3, -2, 22)$

h) $z = e^{xy}$; $(1, 0, 1)$ *and* $(0, 1, 1)$

i) $z = \sin(xy) + 2$; $(1, 0, 2)$ *and* $(0, 5, 2)$

Exercise 2

Find the linear approximation to the function f at the given point. Use this approximation to estimate the given function value.

a) $f(x, y) = xy + x - y$; $(2, 3)$; *estimate* $f(2.1, 2.99)$.

b) $f(x, y) = -x^2 + 2y^2$; $(3, -1)$; *estimate* $f(3.1, -1.04)$

c) $f(x, y, z) = \ln(1 + x + y + 2z)$; $(0, 0, 0)$; *estimate* $f(0.1, -0.2, 0.2)$

Optimization

Exercise 3 *Find all critical points of the following functions.*

a) $f(x, y) = 3x^2 - 4y^2$

b) $f(x, y) = x^2 - 6x + y^2 + 8y$

c) $f(x, y) = \dfrac{x^3}{3} - \dfrac{y^3}{3} + 3xy$

d) $f(x, y) = x^4 - 2x^2 + y^2 - 4y + 5$

e) $f(x, y) = e^{8x^2y^2 - 24x^2 - 8xy^4}$

Exercise 4 *Find the critical points of the following functions. Use the Second Derivative Test to determine (if possible) whether each critical point corresponds to a local maximum, a local minimum, or a saddle point. If the Second Derivative Test is inconclusive, determine the behavior of the function at the critical points.*

a) $f(x,y) = -4x^2 + 8y^2 - 3$

b) $f(x,y) = 4 + 2x^2 + 3y^2$

c) $f(x,y) = x^4 + 2y^2 - 4xy$

d) $f(x,y) = \sqrt{x^2 + y^2 - 4x + 5}$

e) $f(x,y) = 2xye^{-x^2-y^2}$

f) $f(x,y) = \dfrac{x}{1 + x^2 + y^2}$

g) $f(x,y) = x^4 + 4x^2(y-2) + 8(y-1)^2$

h) $f(x,y) = xe^{-x-y}\sin y$, *for* $|x| \leq 2,\ 0 \leq y \leq \pi$

Exercise 5 *A shipping company handles rectangular boxes provided the sum of the height and the girth of the box does not exceed* 240 *cm. (The girth is the perimeter of the smallest side of the box.) Find the dimensions of the box that meets this condition and has the largest volume.*

Exercise 6 *A lidless cardboard box is to be made with a volume of* $4m^3$. *Find the dimensions of the box that requires the least amount of cardboard.*

Exercise 7 *Find the absolute maximum and minimum values of the following functions on the given region R.*

a) $f(x,y) = x^2 + y^2 - 2y + 1; R = (x,y)\colon x^2 + y^2 \leq 4$

b) $f(x,y) = 4 + 2x^2 + y^2; R = (x,y)\colon -1 \leq x \leq 1, -1 \leq y \leq 1$

c) $f(x,y) = 2x^2 - 4x + 3y^2 + 2; R = (x,y)\colon (x-1)^2 + y^2 \leq 1$

d) $f(x,y) = -2x^2 + 4x - 3y^2 - 6y - 1; R = (x,y)\colon (x-1)^2 + (y+1)^2 \leq 1$

e) $f(x,y) = \sqrt{x^2 + y^2 - 2x + 2}; R = (x,y)\colon x^2 + y^2 \leq 4, y \geq 0$

f) $f(x,y) = \dfrac{2y^2 - x^2}{2 + 2x^2y^2}$; *R is the closed region bounded by the lines* $y = x, y = 2x$, *and* $y = 2$.