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Calculus
CSCI 2025
Session 3

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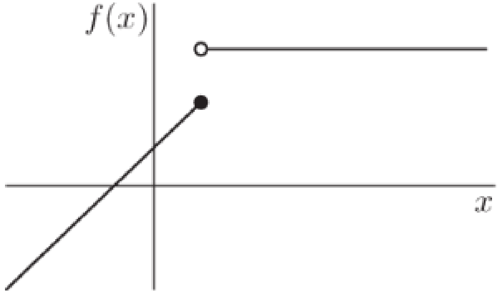
Continuity

What is continuity?

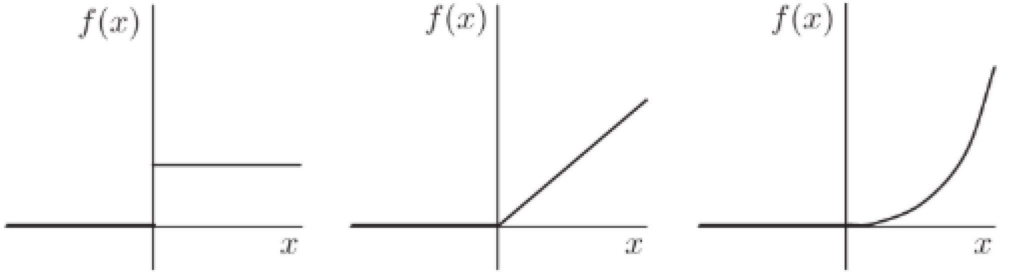
- Start thinking of continuity as a property that the function might have at a point.
- Is $f(x)$ continuous at $x = 1$?



$$f(x) = \begin{cases} x+1 & \text{if } x \leq 1 \\ 3 & \text{if } x > 1 \end{cases}$$

Continuity

Continuous at 0?



$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases} \qquad f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases} \qquad f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

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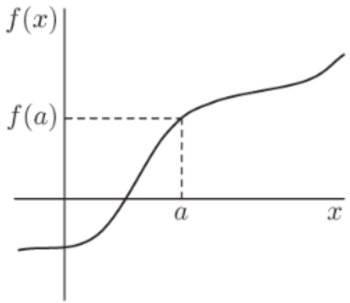
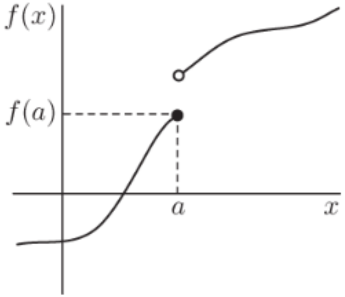
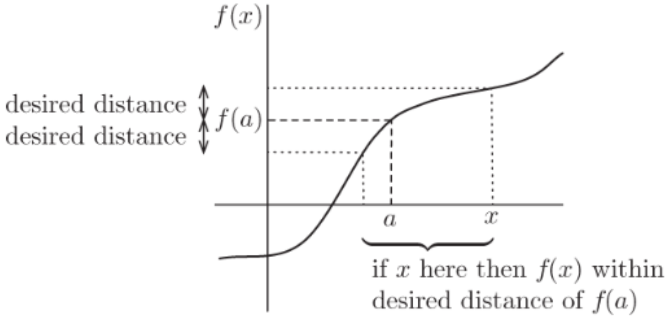
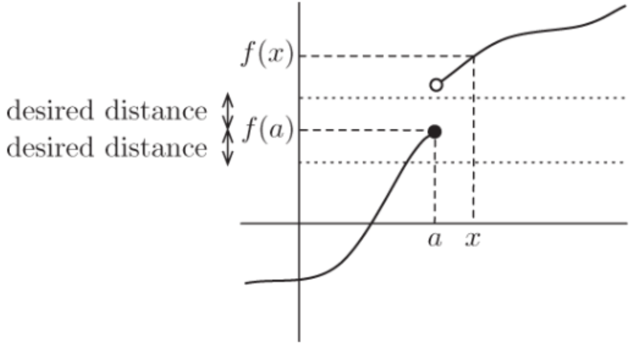
Continuity

Continuous at a?









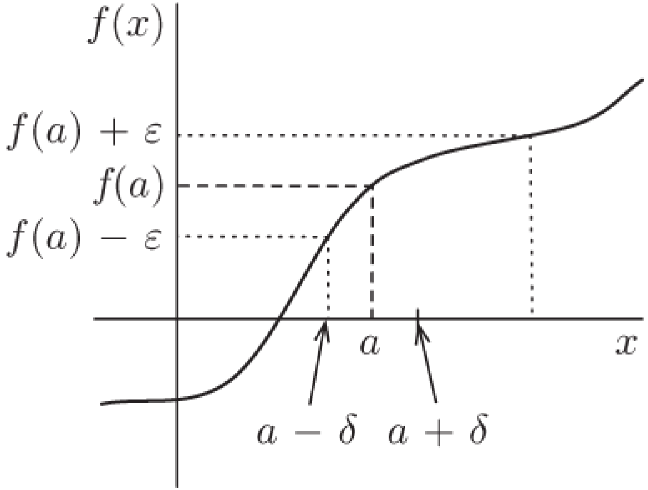
Continuity

Definition

A function $f: \mathbb{R} \to \mathbb{R}$ is *continuous* at $a \in \mathbb{R}$ if and only if

$\forall \varepsilon > 0 \; \exists \delta > 0$ such that if $|x - a| < \delta$ then $|f(x) - f(a)| < \varepsilon$.

- $\forall \varepsilon > 0$: however small epsilon is
- $\exists \delta > 0$: there is a distance delta
- $|x - a| < \delta$: the distance between x and a is less than delta
- $|f(x) - f(a)| < \varepsilon$: the distance between $f(x)$ and $f(a)$ is less than epsilon.



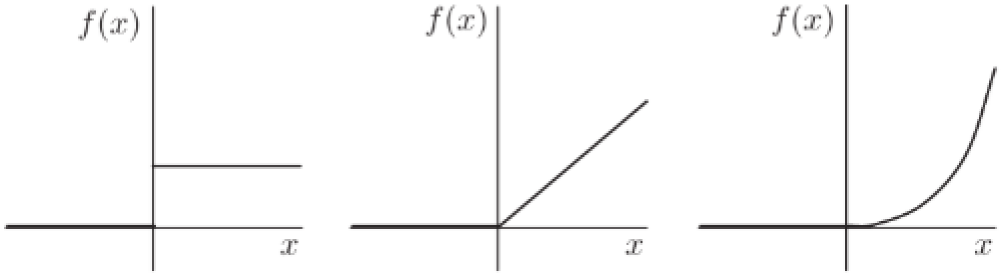
Continuity

Definition

A function $f: \mathbb{R} \to \mathbb{R}$ is *continuous* at $a \in \mathbb{R}$ if and only if $\lim_{x \to a} f(x)$ exists and it is equal to $f(a)$.

Continuity

Continuous at $\mathbf{0}$?



$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases} \qquad f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases} \qquad f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

Continuity

Continuous at $\mathbf{1}$?

$f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = \begin{cases} x+1, & if\ x \leq 1 \\ 3, & if\ x > 1 \end{cases}$

Continuity

Example

$f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = \begin{cases} 2x+1, & x < 3 \\ 5, & x = 3 \\ 6, & x > 3 \end{cases}$.

Find

- $\lim_{x \to 0^+} f(x)$
- $\lim_{x \to 0^-} f(x)$
- $\lim_{x \to 0} f(x)$
- $\lim_{x \to 3^+} f(x)$
- $\lim_{x \to 3^-} f(x)$
- $\lim_{x \to 3} f(x)$

- Is $f(x)$ continuous at 0?
- Is $f(x)$ continuous at 3?

Continuity

Continuity Rules

Theorem

If f and g are continuous at a, then the following functions are also continuous at a.

Assume c is a constant and $n > 0$ is an integer.

- $f \pm g$
- cf
- fg
- f/g, provided $g(a) \neq 0$
- $(f(x))^n$

Continuity

Continuity on an Interval

Definition

A function f is continuous on an interval I if it is continuous at all points of I. If I contains its endpoints, continuity on I means continuous from the right or left at the endpoints.

Continuity on an Interval

Example

Determine the intervals of continuity for

$$f(x) = \begin{cases} x^2 + 1, if\ x \leq 0 \\ 3x + 5, if\ x > 0 \end{cases}$$

Continuity

Continuity of Transcendental Functions

Theorem

The following functions are continuous at all points of their domains.

- Trigonometric: $sin\, x, cos\, x, tan\, x, cot\, x$
- Inverse Trigonometric: $sin^{-1}x, cos^{-1}x, tan^{-1}x, cot^{-1}x,$
- Exponential: b^x, e^x
- Logarithmic: $log_b\, x, lnx$

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Thank you!