Sequences and Series

Exercise 1 *A sequence is given by* $5, \frac{5}{8}, \frac{5}{27}, \frac{5}{64}, \ldots$. *Write down an expression to denote the full sequence.*

Exercise 2 *Write out explicitly the sum*

$$\sum_{k=1}^{4} \frac{1}{(2k+1)(2k+3)}$$

Exercise 3 *a) Write out the first five terms of the sequence* $a_n = \sqrt{n}, n = 1,2,3,\ldots$.

b) Find $\lim_{n\to\infty} a_n$.

Exercise 4 *Find if possible the limit of each of the following sequences:*

a) $a_n = \dfrac{1}{n+1}$

b) $b_n = n^2$

c) $c_n = \dfrac{n-1}{n+1}$

d) $d_n = \left(\dfrac{1}{3}\right)^n$

e) $e_n = \dfrac{2n+3}{4n+2}$

f) $f_n = \dfrac{3n^2+2n-7}{9n^2-7n}$

Exercise 5 *Find the limit of the following sequences or determine that the limit does not exist:*

a) $a_k = \dfrac{k^{12}}{3k^{12}+4}$

b) $b_k = \dfrac{3k^3-1}{2k^3+1}$

c) $c_k = \dfrac{k}{\sqrt{9k^2+1}}$

d) $d_k = tan^{-1}k$

e) $e_k = \sqrt{k^2+1} - k$

f) $f_k = \left(1+\frac{2}{k}\right)^k$

g) $g_k = \sqrt{\left(1+\dfrac{1}{2k}\right)^k}$

h) $h_k = \dfrac{k}{e^k+3k}$

i) $i_k = \left(\dfrac{1}{k}\right)^{\frac{1}{k}}$

j) $j_k = \dfrac{(-1)^k k}{k+1}$

Exercise 6 *Many people take aspirin on a regular basis as a preventive measure for heart disease. Suppose a person takes 80 mg of aspirin every 24 hours. Assume also that aspirin has a half-life of 24 hours; that is, every 24 hours, half of the drug in the blood is eliminated.*

a) Find a recurrence relation for the sequence (d_n) that gives the amount of drug in the blood after the nth dose, where $d_1 = 80$.

b) Using a calculator, determine the limit of the sequence. In the long run, how much drug is in the person's blood?

c) Confirm the result of part (b) by finding the limit of (d_n) directly.

d) Can you express the contribution of each dose as a geometric series and confirm the result of part (b).

Exercise 7 *James begins a savings plan in which he deposits 100 € at the beginning of each month into an account that earns 0.75% interest per month. To be clear, on the first day of each month, the bank adds 0.75% of the current balance as interest, and then James deposits 100 €. Let (B_n) be the loan balance immediately after the nth deposit, where $B_0 = 0$ €.*

a) Write the first five terms of the sequence (B_n).

b) Find a recurrence relation that generates the sequence (B_n).

c) Can you express B_n as a geometric series?

d) How many months are needed to reach a balance of 5 000 €.

Exercise 8 *Determine whether the following sequences converge or diverge, and state whether they are monotonic. Give the limit when the sequence converges.*

a) $(2^{n+1}3^{-n})$

b) $((-2.5)^n)$

c) $(100(-0.003)^n)$

Exercise 9 *Find the limit of the following sequences or state that they diverge.*

a) $a_n = \dfrac{\cos n}{n}$

b) $b_n = \dfrac{\sin 6n}{5n}$

c) $c_n = \dfrac{\sin n}{2^n}$

d) $d_n = \dfrac{\cos(n\frac{\pi}{2})}{\sqrt{n}}$

Exercise 10 *Evaluate each geometric series or state that it diverges.*

a) $\sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k$

b) $1 + \dfrac{1}{\pi} + \dfrac{1}{\pi^2} + \dfrac{1}{\pi^3} + ...$

c) $\sum_{k=1}^{\infty} e^{-2k}$

d) $\sum_{m=2}^{\infty} \dfrac{5}{2^m}$

e) $\sum_{k=3}^{\infty} \frac{3 \cdot 4^k}{7^k}$

f) $\sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k 5^{3-k}$