Exercise Graph Theory

Example 1 *Consider the travel times (in minutes) of Table 1.*

Location (Junction)	**WN**	**E**	**W**	**P**	**L**	**S**	**I**	**B**	**St. M**	**G**	**K**
Wr. Neustadt (A2–S4)	–	20	30	–	–	–	–	–	110	80	–
Eisenstadt (A3–S31)	20	–	20	–	–	–	–	–	–	–	–
Wien–Vösendorf (A2–A21–A23–S1)	30	20	–	40	–	–	–	–	–	–	–
St. Pölten (A1–S33)	–	–	40	–	80	–	–	–	–	–	–
Linz–Voralpenkreuz (A1–A8–A9)	–	–	–	80	–	60	–	–	70	–	–
Salzburg (A1–A10)	–	–	–	–	60	–	110	190	130	–	130
Innsbruck (A12–A13)	–	–	–	–	–	110	–	120	–	–	–
Bregenz (A14)	–	–	–	–	–	190	120	–	–	–	–
St. Michael (A9–S6–S36)	110	–	–	–	70	130	–	–	–	30	90
Graz (A2–A9)	80	–	–	–	–	–	–	–	30	–	80
Klagenfurt (A2–S37)	–	–	–	–	–	130	–	–	90	80	–

Table 1: Travel times in minutes

- *Voluntary: Visit all cities and make a photo of you and the town's landmark.*
- *Draw the corresponding graph.*
- *You want to travel from Wiener Neustadt to Bregenz. Use Dijkstra's algorithm to find the fastest path. No algorithmic shortcuts are allowed. Even if they are obvious. Your algorithm should start in Bregenz.*

Example 2 *Imagine you are responsible for maintaining the occurring streets of Table 1. So you have to find a route which uses all occurring streets at least once. Therefore we are interested in Eulerian paths and circuits.*

Use Table 1 again. Is it possible to find an Eulerian path? What about an Eulerian circuit? Explain your answer.

Can you add some edges (Eulerization) such that an Eulerian circuit is possible? Try to add a little amount of small edges such that the total time for the circuit is not too high. Use Fleury's algorithm to find that circuit.

Example 3 *Imagine you are a salesperson who has to visit all cities of Table 1. Therefore we are interested in Hamiltonian paths and circuits.*

Is it possible to find an Hamiltonian path? If yes, write one down and compute the travel time.

What about an Hamiltonian circuit?

	LON	PAR	BER	MAD	ROM	VIE
LON	–	70	140	180	170	160
PAR	70	–	80	150	120	90
BER	140	80	–	200	110	60
MAD	180	150	200	–	130	190
ROM	170	120	110	130	–	100
VIE	160	90	60	190	100	–

Table 2: Flight times in minutes

Example 4 *Look at Table 2. Draw the corresponding graph. Is it a complete graph? How many Hamiltonian circuits are possible starting in Vienna? What about using a brute force algorithm to find the minimum cost Hamiltonian circuit?*

Use nearest neighbour algorithm to find a good Hamiltonian circuit. Start and end point is Vienna. Compute the corresponding time. Is this an optimal algorithm?

What is the idea of repeated nearest neighbour algorithm?

Finally use sorted edges algorithm to solve the problem again. Is this an optimal algorithm?

Example 5 *You want to find a power grid for the cities of Table 2. Now the entries' unit is kilometre instead of minutes (which makes more sense here). Therefore, every city should be connected but the grid should be as cheep as possible (minimize the length of the power grid).*

Is it necessary to have a complete graph when using Kruskal's algorithm?

Apply the algorithm.

Example 6 *Use the idea of the previous example, but use Table 3 instead of Table 2. It is the same table but some connections are missing.*

	LON	PAR	BER	MAD	ROM	VIE
LON	–	70	140	180	170	160
PAR	70	–	80	150	120	–
BER	140	80	–	200	110	60
MAD	180	150	200	–	–	190
ROM	170	120	110	–	–	–
VIE	160	–	60	190	–	–

Table 3: Pipe length in kilometres

Example 7 *Use the graph of the video "Adjacency Matrix (Kimeswenger)". How many different ways from node A to node D exist in 4 steps? Write down all possible ways. Use the adjacency matrix and its powers to find the solution.*

Do the same for starting node B and final node B.

Finally use B as starting node and C as final node.

Example 8 *There is a simple street map with 4 nodes A, B, C and D.*
You can drive from A to B and vice versa.
You can drive from B to C and vice versa. You can drive from C to D and vice versa.

Draw the graph and compute the adjacency matrix M. Finally compute M^2 and M^3. What information do we get from that matrices?

Example 9 *Do the same as in the previous example. But now there are 2 loops. The first one starts and ends at node A. The second loop starts and ends at loop D.*

Example 10 *Do the same as in the previous example. Again consider the two loops. But now you can only drive from A to B but not from B to A. Further more, you can only drive from C to D but not from D to C. How do we call this kind of graphs?*