

Differential Calculus

Exercise 1 Consider the function $f(x) = 0.1x^2$ and the x -values $x_0 = 2$ and $x_1 = 3$. Calculate the corresponding secant.

Exercise 2 Consider the function $f(x) = x^3$. Calculate the tangent of f in the point $(2, f(2))$.

Exercise 3 Compute the first and the second derivative of

a) $f(x) = x^4$

b) $g(x) = e^x$

c) $h(x) = \ln x$, for $x > 0$

Exercise 4 Compute the derivatives of the following functions:

a) $k(x) = 8 + 2x + 3x^5$

b) $m(x) = 3x + e^x$

c) $n(x) = \ln(x) + 5$, for $x > 0$

Exercise 5 Compute the derivatives of the following functions:

a) $a(x) = (2 + 4x + 2x^4) \cdot e^x$

b) $b(x) = (4 + 2x^4) \cdot (2 + 2x^2)$

c) $c(x) = \frac{8+2x+2x^4}{1+x^2}$

d) $d(x) = \frac{2 \cdot e^x}{2x+4x^2}$

Exercise 6 Compute the derivatives of the following functions:

a) $a(x) = e^{2x}$

b) $b(x) = e^{x+x^2}$

c) $c(x) = \ln(1 + x^2)$

d) $d(x) = 2^x$

Exercise 7 Compute the derivatives of the following functions:

a) $f(x) = \frac{x \cdot e^{2x}}{1+x^2}$

b) $g(x) = \ln\left(2 + e^{1+x+2x^2}\right)$

Exercise 8 Compute the tangent for

$$f(x) = x^3 - 10x^2 + x + 100$$

and $x_0 = 2$. What is the absolute and relative error if we consider $x = 3$ and if we use the tangent instead of f ?

Exercise 9 Lets consider the function $f(x) = x^2 - x^3$ and the x -values $x_0 = -2$ and $x_1 = 2$. Find the corresponding secant.

Exercise 10 Compute the first 3 derivatives of the following functions:

a) $a(x) = 2 + 4x^2 + 2x^5$,

b) $b(x) = x + \frac{2}{x}$,

c) $c(x) = 4^{2x}$

Exercise 11 Compute the first and the second derivatives of the following functions:

a) $a(x) = \frac{x^2-4}{x-2}$,

b) $b(x) = \frac{(2x-3)^2}{(x-1)^2}$,

c) $c(x) = \frac{e^{2x}}{x}$

Exercise 12 Find the tangents of the following functions for

a) $a(x) = 2x^2 + 4$ with $x_0 = 1$,

b) $b(x) = \ln x$ with $x_0 = 1$,

c) $c(x) = \ln(x + 1)$ with $x_0 = 0$,

d) $d(x) = \frac{1}{x}$ with $x_0 = 1$.

Exercise 13 Compute the tangent for

$$f(x) = -2x^3 + 20x^2 - 2x - 200$$

and $x_0 = 2$. What is the absolute and relative error if we consider $x = 3$ and if we use the tangent instead of f ?

Exercise 14 Find all relative minima/maxima and saddle points of

$$x^2 \cdot (x - 1)^3.$$

a) What is the global minimum/maximum, if the domain of interest is \mathbb{R} ?

b) What is the global minimum/maximum, if the domain of interest is the interval $[0,1.5]$?

Exercise 15 Consider the function

$$f(x) = -0.27 + 0.47x - 0.09x^2 + 0.01x^3.$$

Find inflection points if there are any.

Exercise 16 Consider the functions

$$f(x) = x^2 \quad \text{and} \quad g(x) = x^4.$$

Find inflection points if there are any.

Exercise 17 Consider the function

$$f(x) = -960 + 368x - 36x^2 + x^3.$$

Find all x such that $f(x)$ is increasing. Finally find all x such that $f(x)$ is decreasing.

Exercise 18 Find x such that the function

$$f(x) = 10 + 10x^2 - 10x^3 + 2x^4$$

is increasing/decreasing.