Euclidean Algorithm

Exercise 1 *Find the greatest common divisor of*

a) 527 *and* 391

b) 798 *and* 273

c) 1029 *and* 672

using the Euclidean algorithm.

Cryptography

Exercise 2 *Encrypt the message STOP POLLUTION by translating the letters into numbers, applying the given encryption function, and then translating the numbers back into letters.*

a) $f(p) = (p + 4)\,\boldsymbol{mod}\ 26$

b) $f(p) = (p + 21)\,\boldsymbol{mod}\ 26$

c) $f(p) = (17p + 22)\,\boldsymbol{mod}\ 26$

Exercise 3 *Encrypt the message WATCH YOUR STEP by translating the letters into numbers, applying the given encryption function, and then translating the numbers back into letters.*

a) $f(p) = (p + 14)\,\boldsymbol{mod}\ 26$

b) $f(p) = (14p + 21)\,\boldsymbol{mod}\ 26$

c) $f(p) = (-7p + 1)\,\boldsymbol{mod}\ 26$

Exercise 4 *Decrypt these messages that were encrypted using the Caesar cipher.*

a) EOXH MHDQV

b) WHVW WRGDB

c) HDW GLP VXP

Exercise 5 *Decrypt these messages encrypted using the shift cipher* $f(p) = (p+10)\,\boldsymbol{mod}$ 26.

a) CEBBOXNOB XYG

b) LO WI PBSOXN

c) DSWO PYB PEX

Boolean Algebra

Exercise 6 *Find the values of these expressions.*

a) $1 \cdot \overline{0}$

b) $1 + \overline{1}$

c) $\overline{0} \cdot 0$

d) $\overline{(1 + 0)}$

Exercise 7 *Show that* $(1 \cdot 1) + (\overline{0 \cdot 1} + 0) = 1$.

Exercise 8 *Show that* $(\overline{1} \cdot \overline{0}) + (1 \cdot \overline{0}) = 1$.

Exercise 9 *Use a table to express the values of each of these Boolean functions.*

a) $F(x,y,z) = \overline{x}y$

b) $F(x,y,z) = x + yz$

c) $F(x,y,z) = x\overline{y} + \overline{(xyz)}$

d) $F(x,y,z) = x(yz + \overline{y}\overline{z})$

Exercise 10 *Use a table to express the values of each of these Boolean functions.*

a) $F(x,y,z) = \overline{z}$

b) $F(x,y,z) = \overline{x}y + \overline{y}z$

c) $F(x,y,z) = x\overline{y}z + \overline{(xyz)}$

d) $F(x,y,z) = \overline{y}(xz + \overline{x}\ \overline{z})$

Exercise 11 *What values of the Boolean variables x and y satisfy $xy = x + y$?*

Exercise 12 *Show that* ***De Morgan's laws*** *hold in a Boolean algebra. That is, show that for all x and y,* $\overline{(x \vee y)} = \overline{x} \wedge \overline{y}$ *and* $\overline{(x \wedge y)} = \overline{x} \vee \overline{y}$.