

Number Theory

Exercise 1 Does 17 divide each of these numbers?

- a) 68
- b) 84
- c) 357
- d) 1001

Exercise 2 Show that if a, b, c , and d are integers, where $a \neq 0$, such that $a \mid c$ and $b \mid d$, then $ab \mid cd$.

Exercise 3 What are the quotient and remainder when

- a) 44 is divided by 8?
- b) 777 is divided by 21?
- c) -123 is divided by 19?
- d) -1 is divided by 23?
- e) -2002 is divided by 87?
- f) 0 is divided by 17?
- g) 1.234.567 is divided by 1001?
- h) -100 is divided by 101?

Exercise 4 What time does a 12-hour clock read

- a) 80 hours after it reads 11 : 00?
- b) 40 hours before it reads 12 : 00?
- c) 100 hours after it reads 6 : 00?

Exercise 5 Suppose that a and b are integers, $a \equiv 11(\text{mod}19)$, and $b \equiv 3(\text{mod}19)$. Find the integer c with $0 \leq c \leq 18$ such that

- a) $c \equiv 13a(\text{mod}19)$.
- b) $c \equiv 8b(\text{mod}19)$.
- c) $c \equiv a - b(\text{mod}19)$.
- d) $c \equiv 7a + 3b(\text{mod}19)$.
- e) $c \equiv 2a^2 + 3b^2(\text{mod}19)$.
- f) $c \equiv a^3 + 4b^3(\text{mod}19)$.

Exercise 6 Evaluate these quantities:

- a) $-17 \text{mod} 2$
- b) $144 \text{mod} 7$
- c) $-101 \text{mod} 13$

d) $199 \pmod{19}$

Exercise 7 Find the integer a such that

a) $a \equiv 43 \pmod{23}$ and $-22 \leq a \leq 0$

b) $a \equiv 17 \pmod{29}$ and $-14 \leq a \leq 14$

c) $a \equiv -11 \pmod{21}$ and $90 \leq a \leq 110$

Exercise 8 List five integers that are congruent to 4 modulo 12.

Exercise 9 Determine whether each of these integers is prime.

a) 19

d) 101

b) 27

e) 107

c) 93

f) 113

Exercise 10 Find the prime factorization of each of these integers.

a) 39

d) 289

b) 81

e) 729

c) 143

f) 899

Exercise 11 What are the greatest common divisors of these pairs of integers?

a) $3^7 \cdot 5^3 \cdot 7^3, 2^{11} \cdot 3^5 \cdot 5^9$

d) $41 \cdot 43 \cdot 53, 41 \cdot 43 \cdot 53$

b) $11 \cdot 13 \cdot 17, 2^9 \cdot 3^7 \cdot 5^5 \cdot 7^3$

e) $3^{13} \cdot 5^{17}, 2^{12} \cdot 7^{21}$

c) $23^{31}, 23^{17}$

f) 1111, 0

Exercise 12 What is the least common multiple of each pair in the previous exercise?