

## Sets

**Exercise 1** List the members of these sets.

- a)  $\{x \mid x \text{ is a real number such that } x^2 = 1\}$
- b)  $\{x \mid x \text{ is a positive integer less than } 12\}$
- c)  $\{x \mid x \text{ is the square of an integer and } x < 100\}$
- d)  $\{x \mid x \text{ is an integer such that } x^2 = 2\}$

**Exercise 2** Use set builder notation to give a description of each of these sets.

- a)  $\{0, 3, 6, 9, 12\}$
- b)  $\{-3, -2, -1, 0, 1, 2, 3\}$
- c)  $\{m, n, o, p\}$

**Exercise 3** For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

- a) the set of airline flights from New York to New Delhi, the set of nonstop airline flights from New York to New Delhi
- b) the set of people who speak English, the set of people who speak Chinese
- c) the set of flying squirrels, the set of living creatures that can fly

**Exercise 4** Suppose that  $A = \{2,4,6\}$ ,  $B = \{2,6\}$ ,  $C = \{4,6\}$ , and  $D = \{4,6,8\}$ . Determine which of these sets are subsets of which other of these sets.

**Exercise 5** Determine whether each of these statements is true or false.

- a)  $0 \in \emptyset$
- b)  $\emptyset \in \{0\}$
- c)  $\{0\} \subset \emptyset$
- d)  $\emptyset \subset \{0\}$
- e)  $\{0\} \in \{0\}$
- f)  $\{0\} \subset \{0\}$
- g)  $\{\emptyset\} \subseteq \{\emptyset\}$

**Exercise 6** Find the power set of each of these sets, where  $a$  and  $b$  are distinct elements.

- a)  $\{a\}$
- b)  $\{a, b\}$
- c)  $\{\emptyset, \{\emptyset\}\}$

**Exercise 7** How many elements does each of these sets have where  $a$  and  $b$  are distinct elements?

- a)  $\mathcal{P}(\{a, b, \{a, b\}\})$
- b)  $\mathcal{P}(\{\emptyset, a, \{a\}, \{\{a\}\}\})$
- c)  $\mathcal{P}(\mathcal{P}(\emptyset))$

**Exercise 8** Let  $A = \{a, b, c, d\}$  and  $B = \{y, z\}$ . Find

- a)  $A \times B$
- b)  $B \times A$ .

**Exercise 9** What is the Cartesian product  $A \times B$ , where  $A$  is the set of courses offered by the mathematics department at a university and  $B$  is the set of mathematics professors at this university? Give an example of how this Cartesian product can be used.

**Exercise 10** What is the Cartesian product  $A \times B \times C$ , where  $A$  is the set of all airlines and  $B$  and  $C$  are both the set of all cities in France? Give an example of how this Cartesian product can be used.

**Exercise 11** Find  $A^2 = A \times A$  if

- a)  $A = \{0, 1, 3\}$
- b)  $A = \{1, 2, a, b\}$

**Exercise 12** Let  $A$  be the set of students who live within one mile of school and let  $B$  be the set of students who walk to classes. Describe the students in each of these sets.

- a)  $A \cap B$
- b)  $A \cup B$
- c)  $A - B$
- d)  $B - A$

**Exercise 13** Suppose that  $A$  is the set of sophomores at your school and  $B$  is the set of students in discrete mathematics at your school. Express each of these sets in terms of  $A$  and  $B$ .

- a) the set of sophomores taking discrete mathematics in your school
- b) the set of sophomores at your school who are not taking discrete mathematics

- c) the set of students at your school who either are sophomores or are taking discrete mathematics
- d) the set of students at your school who either are not sophomores or are not taking discrete mathematics

**Exercise 14** Let  $A = \{a, b, c, d, e\}$  and  $B = \{a, b, c, d, e, f, g, h\}$ . Find

- a)  $A \cup B$
- b)  $A \cap B$
- c)  $A - B$
- d)  $B - A$

**Exercise 15** Let  $A = \{0, 2, 4, 6, 8, 10\}$ ,  $B = \{0, 1, 2, 3, 4, 5, 6\}$ , and  $C = \{4, 5, 6, 7, 8, 9, 10\}$ . Find

- a)  $A \cap B \cap C$
- b)  $A \cup B \cup C$
- c)  $(A \cup B) \cap C$
- d)  $(A \cap B) \cup C$