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Discrete Mathematics and Algebra
CSCI 2025

Session 3

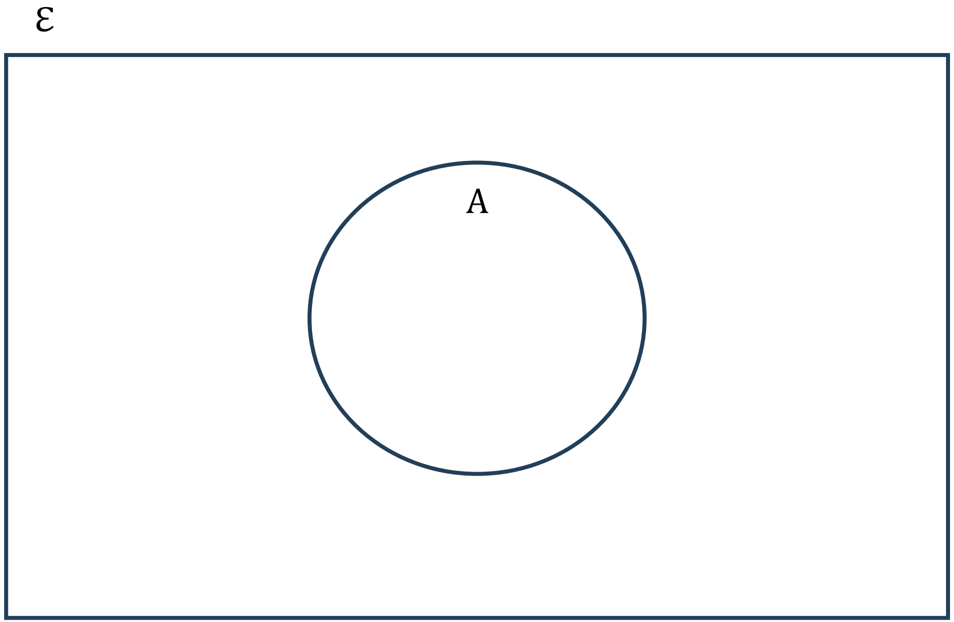
Eirini Kouvela
Institute of Mathematics & Statistics

Overview

- Venn Diagrams
- Number Sets

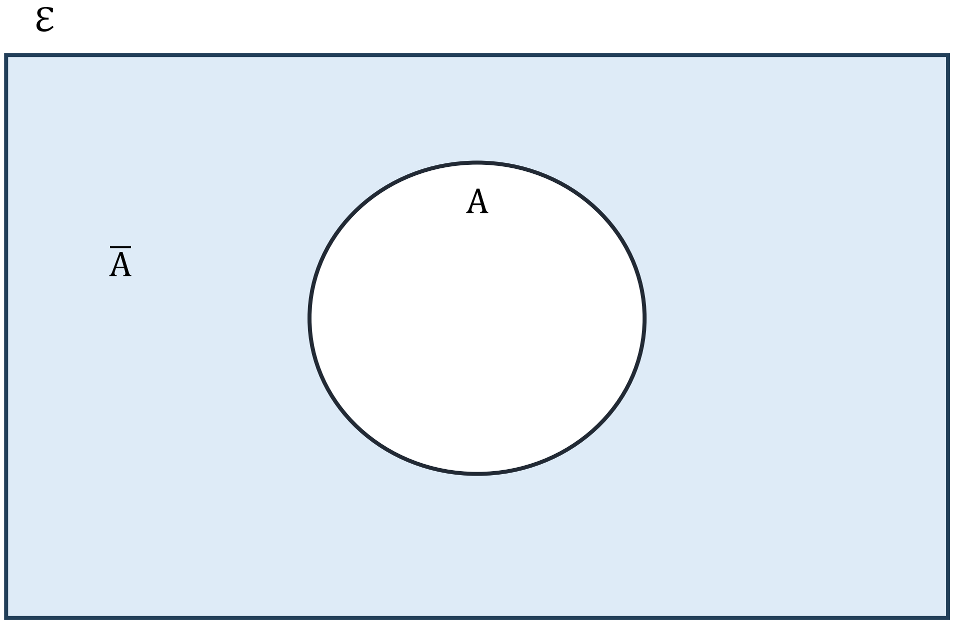
Venn Diagrams

The universal set $\mathcal{E}$ and the set A



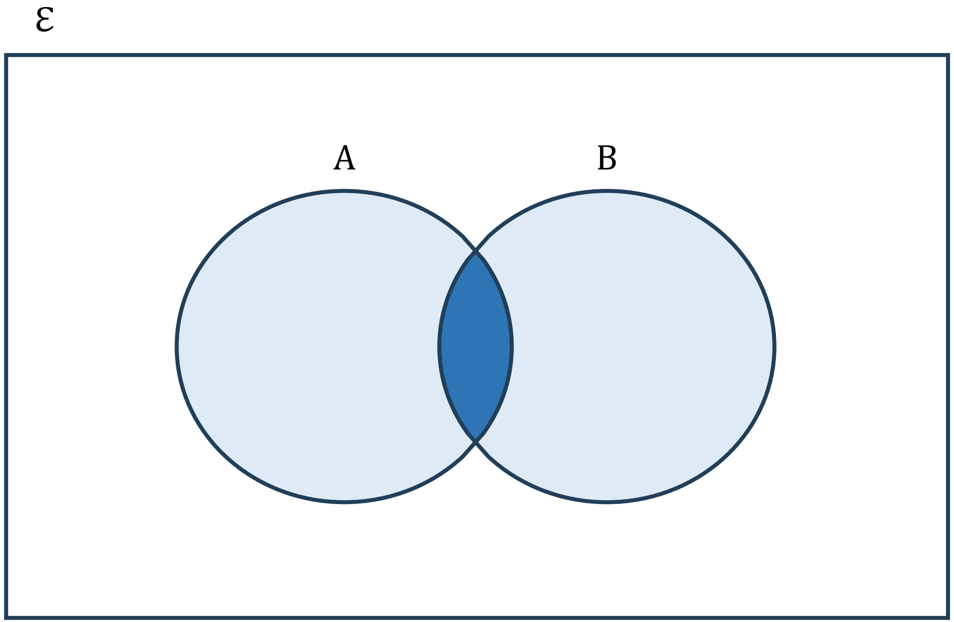
Venn Diagrams

The complement of A, that is $\bar{A}$, is represented by the shaded area



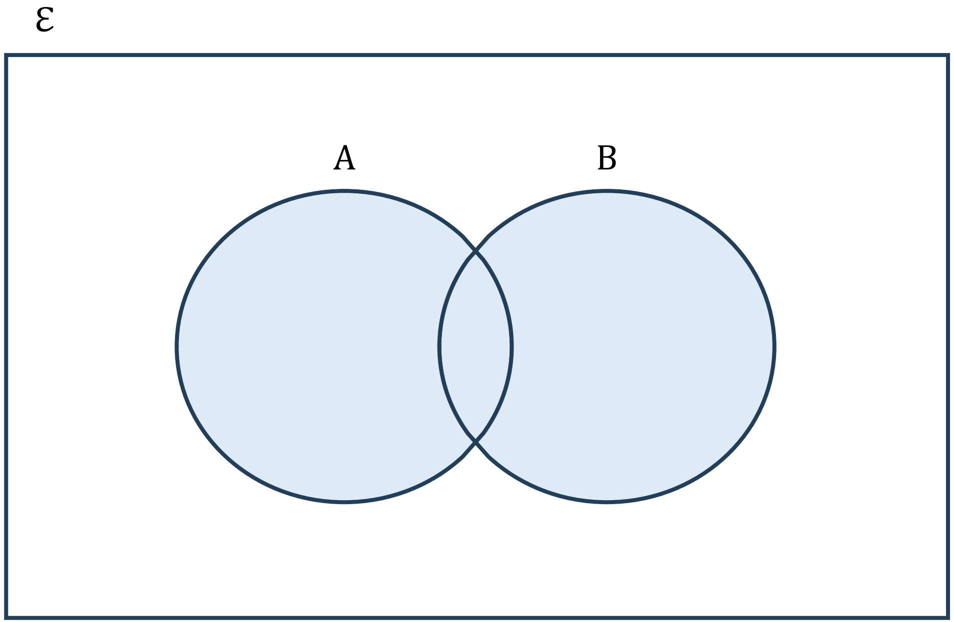
Venn Diagrams

The deeply shaded area represents the set $A \cap B$



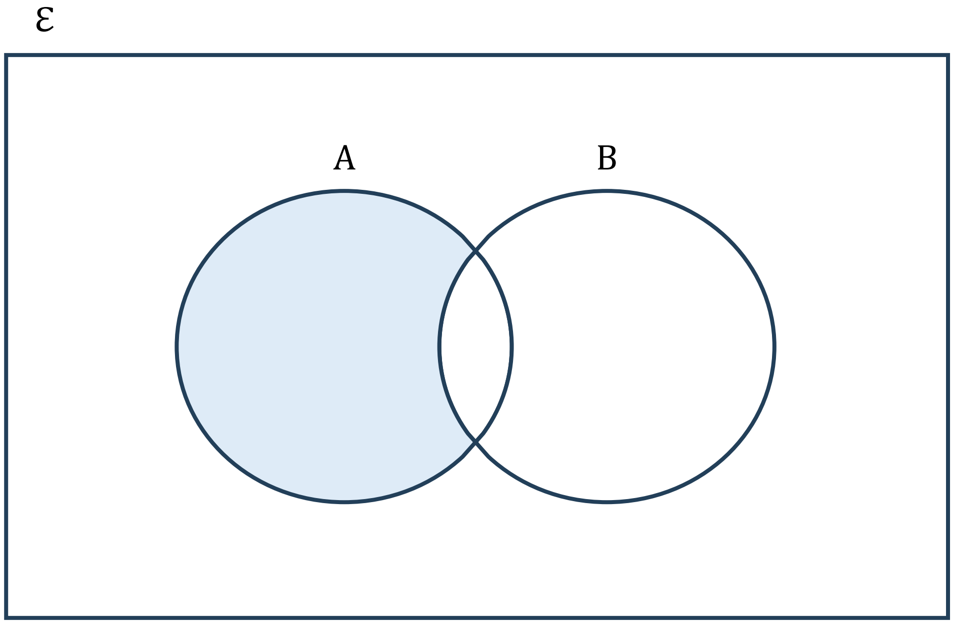
Venn Diagrams

The set $A \cup B$



Venn Diagrams

The set $A - B$ $(A\backslash \mathrm{B})$



Venn Diagrams

Example

A and B are intersecting sets. Represent on a Venn diagram

- $\overline{A \cup B}$
- $\overline{A \cap B}$
- $\overline{A} \cup B$

Number sets

$\mathbb{R}$: Real numbers

$\mathbb{R}$ includes integers and fractions, both positive and negative.

$\mathbb{R} = \{x: -\infty < x < \infty\}$

There are several important subsets of $\mathbb{R}$. The set of all positive numbers is denoted by $\mathbb{R}^+$:

$\mathbb{R}^+ = \{all\ positive\ numbers\} = \{x: x\ is\ greater\ than\ 0\}$

Similarly the set of all negative numbers is denoted by $\mathbb{R}^-$:

$\mathbb{R}^- = \{all\ negative\ numbers\} = \{x: x\ is\ less\ than\ 0\}$

Number sets

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$\mathbb{Z} = \{all\ integers\} = \{\ldots -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3, \ldots\}$

$\mathbb{N} = \{0,\ 1,\ \ 2,\ \ 3,\ \ 4, \ldots\}$

$\mathbb{Q} = \{all\ rational\ numbers\} = \left\{x : x = \frac{p}{q}, p \in \mathbb{Z}, q \in \mathbb{Z}, q\ is\ not\ 0\right\}$

Any number that is not rational is called irrational, for instance $\sqrt{2}$ and π.

$\Pi = \{all\ irrational\ numbers\}$

$\mathbb{Q} \cup \Pi = \mathbb{R}$

$\mathbb{Q} \cap \Pi = \emptyset$

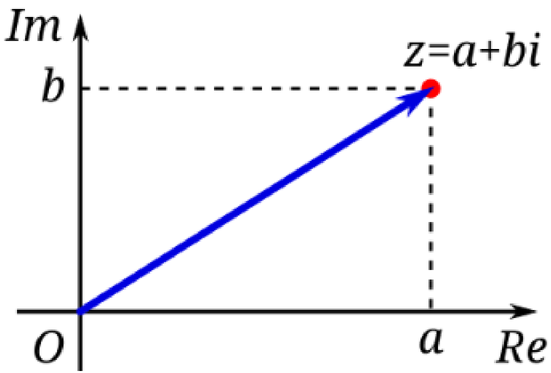
Number sets

ℂ: Complex numbers

A complex number is an element of a number system that extends the real numbers with a specific element denoted i, called the imaginary unit and satisfying the equation $\mathbf{i^2 = -1}$.

Every complex number can be expressed in the form **a+bi**, where a and b are real numbers.

- **a** is the real part
- **b** is called the imaginary part



Number sets

$\mathbb{C}$: Complex numbers

$-4 = 4 \cdot (-1)$

so that

$\sqrt{-4} = \sqrt{4} \cdot \sqrt{-1} = 2 \cdot \sqrt{-1}$

$\sqrt{-4} = 2\mathrm{i}$

$\mathbb{C}$: Complex numbers

Example

State the real and the imaginary parts of the following complex numbers:

- $z = 6i$
- $z = 4$
- $z = 2 + 3i$

$\mathbb{C}$: Complex numbers

Equal complex numbers

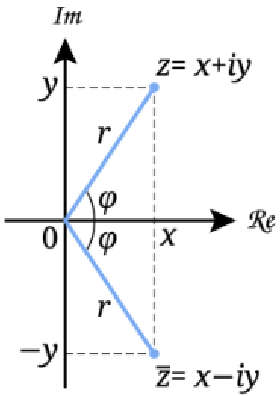
Two complex numbers are equal only when their real parts are equal and their imaginary parts are equal.

So if $a + bi$ is equal to $3 - 4i$ it follows that a must be 3 and b must be -4.

$\mathbb{C}$: Complex numbers

Complex conjugate

If $z = x + yi$, its complex conjugate (reflected on the x-axis), denoted by $\bar{z}$, is $\bar{z} = x - yi$.



Example:

A complex number z_1 is given by $z_1 = x_1 + y_1 i$.

a) State the real and imaginary part of z_1.

b) Write down the complex conjugate of z_1.

$\mathbb{C}$: Complex numbers

Addition and subtraction of complex numbers

If $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$ then

$$z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)i$$
$$z_1 - z_2 = (a_1 - a_2) + (b_1 - b_2)i$$

Example:

If $z_1 = 3 + 11i$ and $z_2 = 18 - 21i$ find

- $z_1 + z_2$
- $z_1 - z_2$

$\mathbb{C}$: Complex numbers

Multiplication of two complex numbers

If $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$ then

$$\begin{aligned} z_1 z_2 &= (a_1 + b_1 i) + (a_2 + b_2 i) \\ &= a_1 a_2 + a_1 b_2 i + b_1 a_2 i + b_1 b_2 i^2 \\ &= (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1) i \end{aligned}$$

Example:

Find $z_1 z_2$ if $z_1 = 3 - 2i$ and $z_2 = 5 + 3i$.

$\mathbb{C}$: Complex numbers

Absolute value

For any complex number $z = a + bi$, the product

$$z \cdot \bar{z} = (a + bi)(a - bi) = a^2 + b^2$$

is a non-negative real number.

The absolute value of a complex number z is:

$$|z| = \sqrt{x^2 + y^2}$$

$\mathbb{C}$: Complex numbers

Absolute value

Example

Compute the absolute value of the following complex numbers:

- $|2|$
- $|2i|$
- $|1 + 2i|$

$\mathbb{C}$: Complex numbers

Division of two complex numbers

$$\frac{z_1}{z_2} = \frac{a_1 + b_1 i}{a_2 + b_2 i}$$

$$= \frac{a_1 + b_1 i}{a_2 + b_2 i} \times \frac{a_2 - b_2 i}{a_2 - b_2 i}$$

$$= \frac{a_1 a_2 + b_1 b_2 + (a_2 b_1 - a_1 b_2) i}{a_2^2 + b_2^2}$$

Example:

If $z_1 = 2 + 9i$ and $z_2 = 5 - 2i$ find $\frac{z_1}{z_2}$.

Thank you!

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