Discrete Mathematics and Algebra
CSCI 2025

Session 2 (Logic)

Eirini Kouvela
Institute of Mathematics & Statistics

FACHHOCHSCHULE
WIENER NEUSTADT
Austrian Network for Higher Education

Logic

Truth Tables

- A compound proposition is made up of simple propositions and the logic symbols ¬ (negation), ∧ (conjunction), ∨ (disjunction), ⊕ (exclusive or), → (implication) and ↔ (bi-implication).
- A truth table lists all possible combinations of thruth values of the simple propositions together with the resulting truth value of the compound proposition.

p	q	$\neg p$	$\neg q$
1	1	0	0
1	0	0	1
0	1	1	0
0	0	1	1

Logic

Truth Tables

- The conjunction, $p \wedge q$, is true only when both p is true and q is true. Otherwise the conjunction is false.

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

Logic

Truth Tables

- The disjunction, $p \vee q$, is true whenever either p is true or q is true or both p and q are true. $p \vee q$ is false only when both p is false and q is also false.

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

Logic

Truth Tables

- The exclusive or, $p \oplus q$, is true when exactly one of p and q is true and is false otherwise.

p	q	$p \oplus q$
1	1	0
1	0	1
0	1	1
0	0	0

Logic

Truth Tables

- The only time that $p \to q$ is false occurs when p is true and q is false.

p	q	$p \to q$
1	1	1
1	0	0
0	1	1
0	0	1

FACHHOCHSCHULE WIENER NEUSTADT
Austrian Network for Higher Education

Logic

Truth Tables

- The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.

p	q	$p \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

Logic

Example

- Construct the truth table for (a) $(\neg p) \land q$ und (b) $p \rightarrow \neg q$.

p	q	$\neg p$	$\neg q$	$(\neg p) \land q$	$p \rightarrow \neg q$
1	1	0	0	0	0
1	0	0	1	0	1
0	1	1	0	1	1
0	0	1	1	0	1

Logic

Example

- Construct the truth table for $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
1	1	0	1	1	1
1	0	1	1	0	0
0	1	0	0	0	1
0	0	1	1	0	0

Logic

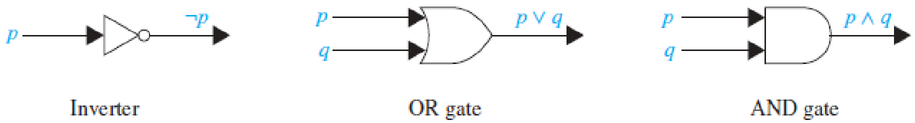
Precedence of logical operators

Operator	Precedence
¬	1
∧ ∨	2 3
→ ↔	4 5

Logic Circuits

Application to Computer Science Problems

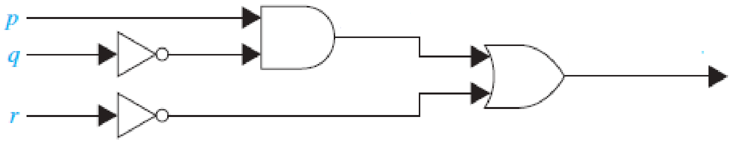
- A **logic circuit** (or **digital circuit**) receives input signals $p_1, p_2, \ldots, p_n$, each a bit [either 0 (off) or 1 (on)], and produces output signals $s_1, s_2, \ldots, s_n$, each a bit.
- Complicated digital circuits can be constructed from three basic circuits, called **gates.**



Logic Circuits

Example

Determine the output for the combinatorial circuit in the following figure.



Logic

Logical equivalences

Double negation law

- $\neg(\neg p) \equiv p$

Commutative laws

- $p \vee q \equiv q \vee p$
- $p \wedge q \equiv q \wedge p$

Associative laws

- $(p \vee q) \vee r \equiv p \vee (q \vee r)$
- $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

Distributive laws

- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

When two compound propositions always have the same truth values, we call them **equivalent.**

FACHHOCHSCHULE WIENER NEUSTADT
Austrian Network for Higher Education

Logic

Logical equivalences

De Morgan‘s laws

- $\neg(p \land q) \equiv \neg p \lor \neg q$
- $\neg(p \lor q) \equiv \neg p \land \neg q$

FACHHOCHSCHULE WIENER NEUSTADT
Austrian Network for Higher Education

Logic

Logical equivalences involving conditional statements

- $p \rightarrow q \equiv \neg p \vee q$
- $p \rightarrow q \equiv \neg q \rightarrow \neg p$
- $p \vee q \equiv \neg p \rightarrow q$
- $p \wedge q \equiv \neg(p \rightarrow \neg q)$
- $\neg(p \rightarrow q) \equiv p \wedge \neg q$
- $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
- $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
- $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
- $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

Logic

Logical equivalences

d: Lisa is in Denmark

e: She is in Europe

$d \rightarrow e$

If Lisa is in Denmark, then she is in Europe.

$\neg e \rightarrow \neg d$

If Lisa is not in Europe, then she is not in Denmark.

FACHHOCHSCHULE
WIENER NEUSTADT
Austrian Network for Higher Education

Thank you!