

# Introduction to R

## Part II/II



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# R - Part II

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# 1 Selected Functions from the dplyr Package

## 1.1 Required Packages

Some very useful packages for data wrangling are

- dplyr
- tibble
- tidyr

I recommend installing the package tidyverse using `install.packages("tidyverse")`, which contains these packages and some more (e.g. ggplot2):

```
> library(tidyverse)
— Attaching packages — tidyverse 1.3.2 —
✓ ggplot2 3.3.6      ✓ purrr 0.3.4
✓ tibble 3.1.8       ✓ dplyr 1.0.9
✓ tidyr 1.2.0        ✓ stringr 1.4.0
✓ readr 2.1.2        ✓ forcats 0.5.2
— Conflicts — tidyverse_conflicts() —
✗ dplyr::filter() masks stats::filter()
✗ dplyr::lag() masks stats::lag()
```

## 1.2 Sample Data

We use the following data.frame:

```
rm(list=ls())
set.seed(123)
var1 <- round(c(rnorm(n=30, mean=30, sd=2), rnorm(n=30, mean=35, sd=2)),3)
var2 <- var1 + 2
set.seed(456)
var3 <- round(rchisq(n=60, df=3),3)
var4 <- rep(c("A","B"), 30)
var5 <- sample(c("X","Y","Z"), 60, replace=TRUE)
mydata <- data.frame(var1, var2, var3, var4, var5)
print(head(mydata,3))
```

```
      var1  var2  var3 var4 var5
1 28.879 30.879 0.215   A   X
2 29.540 31.540 4.638   B   Y
3 33.117 35.117 1.225   A   Z
```

## 1.3 Advanced DM-Operations

### Calculating New Variables

| Function                                                                  | Description                                                              |
|---------------------------------------------------------------------------|--------------------------------------------------------------------------|
| <code>dplyr::mutate(X, newvar1=formula1, newvar2=formula2, ...)</code>    | Adds new variables to a data.frame <b>X</b> and preserves existing ones. |
| <code>dplyr::transmute(X, newvar1=formula1, newvar2=formula2, ...)</code> | Computes new columns in <b>X</b> and drops existing columns.             |

### 💡 What's wrong with transform?

Nothing. A similar base-R function that resembles `mutate` is `transform`. The advantage of `dplyr::mutate` is the ability that a newly created variable is “immediately available” and can be used in the formula for the next variable, which you want to create.

### Example: Calculating New Variables Using `dplyr::transmute`

Note that every variable you do not “mention” in the function call, is deleted:

```
X <- dplyr::transmute(mydata, var1, var6=exp(var1), var7=sin(var2))
print(head(X,1))
```

```
      var1      var6      var7
1 28.879 3.483296e+12 -0.5114974
```

### Selecting Variables

| Function                           | Description                                             |
|------------------------------------|---------------------------------------------------------|
| <code>dplyr::select(X, ...)</code> | Selects variables in a <code>data.frame</code> <b>X</b> |

- Variables can be addressed by using their (unquoted) names (not their positions).
- The colon (`:`) selects a range of consecutive variables.
- The exclamation mark (`!`) takes the complement of a set of variables.
- Useful *selection helpers* are `starts_with`, `ends_with` and `contains`.
- The function `select` can also be used for rearranging variables (without dropping any of them).

### Using the Pipe Operator (`%>%`)

#### ⚠ The Pipe Operator `%>%`

The pipe operator (originally from the `magrittr`-package) pipes an object forward into a function or call expression, i.e. you can rewrite a function call `f(x)` as `x %>% f`, e.g.:

```
library(magrittr)
```

```
x = 7
```

```
x %>% sin
```

```
[1] 0.6569866
```

I personally recommend **to avoid** using this notation.

### Sorting Values

| Function                                        | Description                                                                                                                          |
|-------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------|
| <code>dplyr::arrange(X, var1, var2, ...)</code> | Returns a sorted <code>data.frame</code> <b>X</b> . Use <code>desc(var)</code> to sort by a variable <b>var</b> in descending order. |

## Finding the “Top n” or “Bottom n” Observations

| Function                                      | Description                                                          |
|-----------------------------------------------|----------------------------------------------------------------------|
| <code>dplyr::top_n(x, n, wt)</code>           | Selects top n (or bottom n with negative value for n) rows by value. |
| <code>dplyr::slice_min(X, order_by, n)</code> | Rows with the smallest values of a variable.                         |
| <code>dplyr::slice_max(X, order_by, n)</code> | Rows with the largest values of a variable.                          |

## Selecting Observations

| Function                                 | Description                                                                                             |
|------------------------------------------|---------------------------------------------------------------------------------------------------------|
| <code>dplyr::slice_head(X, n)</code>     | Selects the first rows of your data <b>X</b> (use with an integer <b>n</b> or a fraction <b>prop</b> ). |
| <code>dplyr::slice_tail(X, n)</code>     | Selects the last rows of your data <b>X</b> (use with an integer <b>n</b> or a fraction <b>prop</b> ).  |
| <code>dplyr::slice_sample(X, n)</code>   | Randomly selects observations from <b>X</b> (use with an integer <b>n</b> or a fraction <b>prop</b> ).  |
| <code>dplyr::filter(X, condition)</code> | Selects observations from <b>X</b> based on a logical condition.                                        |

Useful operators for `dplyr::filter` are:

- Classical comparison operators (`>`, `==`, etc.)
- `&` (and), `|` (or), `!` (not) and `xor` (exclusive or)
- `is.na` (for missing values) `dplyr::between()` and `dplyr::near()`

## Creating Summaries

| Function                                     | Description                                                                                           |
|----------------------------------------------|-------------------------------------------------------------------------------------------------------|
| <code>tibble::as_tibble(X)</code>            | Converts (usually a <code>data.frame</code> <b>X</b> ) to a <i>tibble</i> .                           |
| <code>tibble::tibble(col1, col2, ...)</code> | Builds a <i>tibble</i> from individual columns.                                                       |
| <code>dplyr::group_by(X, groupvar)</code>    | Groups data in <b>X</b> , returns a “grouped table”, i.e. all operations will be performed groupwise. |
| <code>dplyr::ungroup(X)</code>               | Undo the grouping.                                                                                    |
| <code>dplyr::summarize(X, ...)</code>        | Summarizes your data (see example).                                                                   |

### Example: Creating (Grouped) Summaries

```
X <- mydata
head(X, 4)
```

|   | var1   | var2   | var3  | var4 | var5 |
|---|--------|--------|-------|------|------|
| 1 | 28.879 | 30.879 | 0.215 | A    | X    |
| 2 | 29.540 | 31.540 | 4.638 | B    | Y    |
| 3 | 33.117 | 35.117 | 1.225 | A    | Z    |
| 4 | 30.141 | 32.141 | 0.826 | B    | X    |

```
X <- mydata
X <- dplyr::group_by(X, var5)
result1 <- dplyr::summarize(X, MEANV1 = mean(var1), SDV1=sd(var1), COUNT=dplyr::n())
print(result1)
```

```
# A tibble: 3 x 4
  var5 MEANV1 SDV1 COUNT
  <chr> <dbl> <dbl> <int>
1 X      32.2  3.05     25
2 Y      32.6  3.34     19
3 Z      33.5  3.63     16
```

Now undo the grouping and calculate the summary again:

```
X <- dplyr::ungroup(X)
result2 <- dplyr::summarize(X, MEANV1 = mean(var1), SDV1=sd(var1), COUNT=dplyr::n())
print(result2)
```

```
# A tibble: 1 x 3
  MEANV1 SDV1 COUNT
  <dbl> <dbl> <int>
1  32.6  3.29     60
```

## Joins in R

### Joins - Mutating Joins

The following *joins* are **mutating joins**. They combine variables from two tables:

| Function                                 | Description |
|------------------------------------------|-------------|
| <code>dplyr::inner_join(x, y, by)</code> | Inner Join  |
| <code>dplyr::left_join(x, y, by)</code>  | Left Join   |
| <code>dplyr::right_join(x, y, by)</code> | Right Join  |
| <code>dplyr::full_join(x, y, by)</code>  | Full Join   |
| <code>dplyr::cross_join(x, y)</code>     | Cross Join  |

### Joins - Filtering Joins

The following *joins* are **filtering joins**. They keep cases from the “left” table:

| Function                                | Description                                                                                                                                                 |
|-----------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>dplyr::semi_join(x, y, by)</code> | Returns all rows from the left table, where there are matching variables in the right table, keeping just columns from the left table (without duplicates). |
| <code>dplyr::anti_join(x, y, by)</code> | Returns all rows from the left table where there are not matching values in the right table, keeping just columns from the left table.                      |

### Sample Data for the Joins

```
t1v1 <- c(1,2,2,3)
t1v2 <- c("a","b","c","d")
t1 <- data.frame(t1v1, t1v2)

t2v1 <- c(1,2,4)
t2v2 <- c("A","B","C")
t2 <- data.frame(t2v1, t2v2)
```

```
print(t1)

  t1v1 t1v2
1     1    a
2     2    b
3     2    c
4     3    d

print(t2)
```

```
  t2v1 t2v2
1     1    A
2     2    B
3     4    C
```

### Example: Left Inner Join

```
# left (inner) join
X1 <- dplyr::left_join(x=t1, y=t2, by=c("t1v1"="t2v1"))
X1

  t1v1 t1v2 t2v2
1     1    a    A
2     2    b    B
3     2    c    B
4     3    d <NA>
```

### Example: Inner Join

```
# inner join
X2 <- dplyr::inner_join(x=t1, y=t2, by=c("t1v1"="t2v1"))
X2

  t1v1 t1v2 t2v2
1     1    a    A
2     2    b    B
3     2    c    B
```

### Example: Right Join

```
# right join
X2 <- dplyr::right_join(x=t1, y=t2, by=c("t1v1"="t2v1"))
X2
```

|   | t1v1 | t1v2 | t2v2 |
|---|------|------|------|
| 1 | 1    | a    | A    |
| 2 | 2    | b    | B    |
| 3 | 2    | c    | B    |
| 4 | 4    | <NA> | C    |

### Example: Full Join

```
# full join
X2 <- dplyr::full_join(x=t1, y=t2, by=c("t1v1"="t2v1"))
X2
```

|   | t1v1 | t1v2 | t2v2 |
|---|------|------|------|
| 1 | 1    | a    | A    |
| 2 | 2    | b    | B    |
| 3 | 2    | c    | B    |
| 4 | 3    | d    | <NA> |
| 5 | 4    | <NA> | C    |

### Example: Cross Join

```
# cross join
X2 <- dplyr::cross_join(x=t1, y=t2)
head(X2, 8)
```

|   | t1v1 | t1v2 | t2v1 | t2v2 |
|---|------|------|------|------|
| 1 | 1    | a    | 1    | A    |
| 2 | 1    | a    | 2    | B    |
| 3 | 1    | a    | 4    | C    |
| 4 | 2    | b    | 1    | A    |
| 5 | 2    | b    | 2    | B    |
| 6 | 2    | b    | 4    | C    |
| 7 | 2    | c    | 1    | A    |
| 8 | 2    | c    | 2    | B    |

## 2 Selected Functions from the tidyr Package

### 2.1 Create Sample Data (a tibble):

```
id <- c(1,2,3,4,5)
vname <-c("Alfred", "Beate", "Christian", "Doris", "Eva")
height <- 171:175
weight <- 72:68
mydata <- tibble::tibble(id, vname, height, weight)
print(mydata)
```

```
# A tibble: 5 x 4
      id vname      height weight
  <dbl> <chr>      <int>  <int>
1     1 Alfred      171     72
2     2 Beate      172     71
3     3 Christian  173     70
4     4 Doris      174     69
5     5 Eva        175     68
```

### 2.2 Using tidyr::gather

With gather we can change a table to *long format*:

```
X1 <- tidyr::gather(mydata, key="Attribute", value="Value", height, weight)
head(X1,7)
```

```
# A tibble: 7 x 4
      id vname      Attribute Value
  <dbl> <chr>      <chr>      <int>
1     1 Alfred      height      171
2     2 Beate      height      172
3     3 Christian  height      173
4     4 Doris      height      174
5     5 Eva        height      175
6     1 Alfred      weight       72
7     2 Beate      weight       71
```

## 2.3 Using tidyr::spread

The function spread is the opposite of gather - we can convert a table back to *wide format*:

```
X2 <- tidyr::spread(X1, key="Attribute", value="Value")
head(X2, 2)
```

```
# A tibble: 2 x 4
      id vname  height weight
  <dbl> <chr>   <int>   <int>
1     1  Alfred    171     72
2     2   Beate    172     71
```

## 2.4 Using tidyr::unite

The function unite unites multiple columns into one column:

```
X3 <- mydata
X3 <- tidyr::unite(mydata, col="alltogether", sep=";")
head(X3, 2)
```

```
# A tibble: 2 x 1
  alltogether
  <chr>
1 1;Alfred;171;72
2 2;Beate;172;71
```

## 2.5 Using tidyr::separate

The opposite of the function unite is separate:

```
X4 <- tidyr::separate(data=X3, col=alltogether, into=c("id", "name", "height", "weight"))
head(X4, 2)
```

```
# A tibble: 2 x 4
      id  name  height weight
  <chr> <chr>   <chr>   <chr>
1 1    Alfred 171     72
2 2    Beate 172     71
```

## 2.6 Handling Missing Values

The following functions from the tidyr package will be useful if you deal with missing values:

### Useful Functions

| Function                            | Description                                                           |
|-------------------------------------|-----------------------------------------------------------------------|
| drop_na(X)                          | Drops rows where any column has missing values.                       |
| fill(X, variable, .direction)       | Fills missing values in selected columns with next or previous entry. |
| replace_na(X, list(var=value, ...)) | Replaces NAs with specified values.                                   |

## Examples

```
x <- c(1, 2, NA, 4, 5)
y <- c(1, NA, 3, 4, 5)
z <- c("group1", NA, NA, "group2", NA)
Z <- data.frame(x, y, z)
Z
```

|   | x  | y  | z      |
|---|----|----|--------|
| 1 | 1  | 1  | group1 |
| 2 | 2  | NA | <NA>   |
| 3 | NA | 3  | <NA>   |
| 4 | 4  | 4  | group2 |
| 5 | 5  | 5  | <NA>   |

### Example: fill

```
Z1 <- tidyr::fill(Z, z, .direction="down")
Z1
```

|   | x  | y  | z      |
|---|----|----|--------|
| 1 | 1  | 1  | group1 |
| 2 | 2  | NA | group1 |
| 3 | NA | 3  | group1 |
| 4 | 4  | 4  | group2 |
| 5 | 5  | 5  | group2 |

### Example: drop\_na

```
Z2 <- tidyr::drop_na(Z)
Z2
```

|   | x | y | z      |
|---|---|---|--------|
| 1 | 1 | 1 | group1 |
| 2 | 4 | 4 | group2 |

### Example: replace\_na

```
Z3 <- tidyr::replace_na(Z, list(x=0, y=0, z="no data"))
Z3
```

|   | x | y | z       |
|---|---|---|---------|
| 1 | 1 | 1 | group1  |
| 2 | 2 | 0 | no data |
| 3 | 0 | 3 | no data |
| 4 | 4 | 4 | group2  |
| 5 | 5 | 5 | no data |

## 3 Statistics

### 3.1 Required Packages

- stats (no need to install)
- cluster - for cluster analysis

### 3.2 Sample Data

We will create a sample data.frame:

```
set.seed(123)
var1 <- round(c(rnorm(n=30, mean=30, sd=2), rnorm(n=30, mean=35, sd=2)),3)
var2 <- var1 + 2
set.seed(456)
var3 <- round(rchisq(n=60, df=3),3)
var4 <- rep(c("A","B"), 30)
var5 <- sample(c("X","Y","Z"), 60, replace=TRUE)
mydata <- data.frame(var1, var2, var3, var4, var5)
print(head(mydata,5))
```

|   | var1   | var2   | var3  | var4 | var5 |
|---|--------|--------|-------|------|------|
| 1 | 28.879 | 30.879 | 0.215 | A    | X    |
| 2 | 29.540 | 31.540 | 4.638 | B    | Y    |
| 3 | 33.117 | 35.117 | 1.225 | A    | Z    |
| 4 | 30.141 | 32.141 | 0.826 | B    | X    |
| 5 | 30.259 | 32.259 | 0.745 | A    | X    |

We have 60 rows and five variables:

```
print(dim(mydata))
```

```
[1] 60  5
```

### 3.3 Statistical Tests

The table below lists the most important statistical tests.

- To find out more about their details, use the help function, e.g. `help(binom.test)` or the abbreviated version, e.g. `?binom.test`.
- The keyword `formula` appears in many functions (e.g. for the T-Test) - it is usually an expression similar to `var1 ~ var2` where **var1** is the “outcome” and **var2** is the “independent” (or sometimes grouping) variable.
- It is a good idea to **save** your output from a test in a variable (which is usually a list or a list-like object).

### 3.4 Statistical Tests - Overview

#### T-Tests

| Test                                               | Description                                                                                |
|----------------------------------------------------|--------------------------------------------------------------------------------------------|
| <code>t.test(x, mu, alternative)</code>            | Performs a <b>one sample T-Test</b>                                                        |
| <code>t.test(formula, data)</code>                 | Performs a <b>Welch-Test</b>                                                               |
| <code>t.test(formula, data, var.equal=TRUE)</code> | Performs a <b>T-Test for independent samples</b> .                                         |
| <code>t.test(x, y, paired=TRUE)</code>             | Performs a <b>T-Test for dependent samples</b> (i.e. repeated measurements).               |
| <code>pairwise.t.test(x, g)</code>                 | Calculate pairwise comparisons between group levels with corrections for multiple testing. |

#### Linear Modelling, ANOVA and Related Functions

| Test                                     | Description                                                                                                                             |
|------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>lm(formula, data)</code>           | Fits linear models.                                                                                                                     |
| <code>predict.lm(object, newdata)</code> | Returns predicted values based on linear model object. The linear model object is obtained from the <code>lm</code> -function.          |
| <code>anova(object)</code>               | Computes analysis of variance tables. The argument <code>object</code> is a linear model (obtained from the <code>lm</code> -function). |
| <code>TukeyHSD(x)</code>                 | Computes Tukey Honest Significant Differences. The argument <code>x</code> is an ANOVA-Table.                                           |

#### Tests for Checking the Model Assumptions

Some statistical tests require a normal distribution in each group and equal variances among the groups.

| Test                                      | Description                                                                                          |
|-------------------------------------------|------------------------------------------------------------------------------------------------------|
| <code>shapiro.test(x)</code>              | Performs the <b>Shapiro-Wilk Test</b> of normality.                                                  |
| <code>bartlett.test(formula, data)</code> | Performs <b>Bartlett's Test</b> of the null that the variances in each group (samples) are the same. |

| Test                                     | Description                                                                                                                                                          |
|------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>ks.test(x, y)</code>               | Performs a one sample <b>Kolmogorov-Smirnov Test</b> . Here, x is the data and y a distribution (such as <code>pnorm</code> to test against the normal distribution) |
| <code>ks.test(x, y)</code>               | Performs a two-sample <b>Smirnov Test</b> that x and y were drawn from the same <i>continuous</i> distribution.                                                      |
| <code>fligner.test(formula, data)</code> | Performs a <b>Fligner-Killeen (median) Test</b> of the null that the variances in each of the groups (samples) are the same.                                         |

### Nonparametric Tests

| Test                                                                               | Description                                                                                                                                                                                |
|------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>binom.test(x, n, p, alternative, conf.level)</code>                          | Performs an exact test ( <b>binomial test</b> ) of a simple null hypothesis about the probability of success in a <b>Bernoulli Experiment</b> .                                            |
| <code>wilcox.test(formula, data)</code><br><code>oneway.test(formula, data)</code> | <b>Wilcoxon</b> or <b>Mann-Whitney-Test</b><br><b>Test for Equal Means</b> in a One-Way Layout (equal variances not necessarily assumed).                                                  |
| <code>kruskal.test(formula, data)</code>                                           | Performs a <b>Kruskal-Wallis Rank Sum Test</b> .                                                                                                                                           |
| <code>chisq.test(x)</code>                                                         | Test for contingency in a frequency table.                                                                                                                                                 |
| <code>chisq.test(x, p, rescale.p)</code>                                           | Compares an empirical with a theoretical distribution. The values in <b>p</b> must be probabilities (unless you specify <code>rescale.p=TRUE</code> ) and of the same length as your data. |
| <code>friedman.test(formula, data)</code>                                          | Performs a <b>Friedman Rank Sum Test</b> with unreplicated blocked data.                                                                                                                   |
| <code>fisher.test(x)</code>                                                        | Performs <b>Fisher's Exact Test</b> for testing the null of independence of rows and columns in a contingency table with fixed marginals.                                                  |
| <code>mood.test(formula, data)</code>                                              | Performs <b>Mood's Two-Sample Test</b> for a difference in scale parameters.                                                                                                               |
| <code>ansari.test(formula, data)</code>                                            | Performs the <b>Ansari-Bradley Two-Sample Test</b> for a difference in scale parameters.                                                                                                   |
| <code>nls(formula, data, ...)</code>                                               | Determine the nonlinear (weighted) least-squares estimates of the parameters of a nonlinear model.                                                                                         |

## Advanced Statistics

| Function                                      | Description                                                                                                                                                                                                                                                                                                                                      |
|-----------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>clara(x,k,metric,stand)</code>          | <b>Clustering Large Applications.</b> You need the <code>cluster</code> package. Here, <b>x</b> is your data, <b>k</b> the number of clusters, <b>metric</b> the metric used to calculate the distances between the objects, <b>stand</b> a logical value (i.e. TRUE or FALSE) that will indicate if your variables shall be standardized first. |
| <code>glm(formula, data, ...)</code>          | <b>Generalized Linear Models</b> , e.g. logistic regression or poisson regression.                                                                                                                                                                                                                                                               |
| <code>predict.glm(object, newdata,...)</code> | Forecasting with a generalized linear model.                                                                                                                                                                                                                                                                                                     |
| <code>princomp(x,...)</code>                  | <b>Principal Components Analysis.</b>                                                                                                                                                                                                                                                                                                            |

### 3.5 Examples

#### Example 1 - Binomial Test

We use the data.frame *mydata*. The outcome “X” should be counted as a success. We want to test if the probability for success is greater than 1/3:

```
binom.test(sum(X$var5=="X"), nrow(X),  
p=1/3, alternative="greater")
```

Exact binomial test

```
data:  sum(X$var5 == "X") and nrow(X)  
number of successes = 25, number of trials = 60, p-value = 0.1101  
alternative hypothesis: true probability of success is greater than 0.3333333  
95 percent confidence interval:  
 0.3087114 1.0000000  
sample estimates:  
probability of success  
 0.4166667
```

## Example 2 - Comparing an Empirical with a Theoretical Distribution

We want to know if the variable *var4* from the data.frame *mydata* follows a discrete uniform distribution, i.e. if the values “X”, “Y” and “Z” have the same probability:

```
X <- mydata
table(mydata$var5)

  X  Y  Z
25 19 16

chisq.test(x = c(25, 19, 16), p=c(1,1,1), rescale.p=TRUE)
```

Chi-squared test for given probabilities

```
data:  c(25, 19, 16)
X-squared = 2.1, df = 2, p-value = 0.3499
```

## Example 3 - $\chi^2$ for Contingency Tables

```
tab <- table(mydata$var4, mydata$var5)
tab
```

|   | X  | Y  | Z |
|---|----|----|---|
| A | 11 | 11 | 8 |
| B | 14 | 8  | 8 |

```
chisq.test(tab)
```

Pearson's Chi-squared test

```
data:  tab
X-squared = 0.83368, df = 2, p-value = 0.6591
```

## Example 4 - ANOVA

We will make use of our sample data again to test if the means of *var1* differ among the groups defined in the variable *var5* using ANOVA (if it is possible). This is our data:

```
X <- mydata
print(head(X,2))
```

```
      var1  var2  var3 var4 var5
1 28.879 30.879 0.215    A    X
2 29.540 31.540 4.638    B    Y
```

Now check the assumptions:

```
# 1) assumption: equal variances
bartlett.test(var1 ~ var5, data=X)
```

Bartlett test of homogeneity of variances

data: var1 by var5

Bartlett's K-squared = 0.57779, df = 2, p-value = 0.7491

```
# 2) assumption: normality
```

```
doBy::summaryBy(var1 ~ var5, data=X, FUN=shapiro.test)[ , 1:4]
```

|   | var5 | var1.statistic | var1.p.value | var1.method                 |
|---|------|----------------|--------------|-----------------------------|
| 1 | X    | 0.9570081      | 0.3581135    | Shapiro-Wilk normality test |
| 2 | Y    | 0.9465947      | 0.34527      | Shapiro-Wilk normality test |
| 3 | Z    | 0.9398446      | 0.3471144    | Shapiro-Wilk normality test |

We calculate ANOVA using `lm()` and output the result in a well-known ANOVA-table:

```
anova(lm(var1 ~ var5, data=X))
```

Analysis of Variance Table

Response: var1

|           | Df | Sum Sq | Mean Sq | F value | Pr(>F) |
|-----------|----|--------|---------|---------|--------|
| var5      | 2  | 16.82  | 8.4102  | 0.7715  | 0.4671 |
| Residuals | 57 | 621.39 | 10.9016 |         |        |

## Example 5 - Logistic Regression

```
X$outcome <- ifelse(X$var4=="A",1,0)
print(head(X,2))
```

```
      var1   var2  var3 var4 var5 outcome
1 28.879 30.879 0.215    A    X        1
2 29.540 31.540 4.638    B    Y        0
```

```
logisticmodel <- glm(outcome ~ var3, data=X, family=binomial(link="logit"))
print(confint(logisticmodel))
```

Waiting for profiling to be done...

```
              2.5 %    97.5 %
(Intercept) -0.7009819 0.8991228
var3         -0.2319846 0.1653252
```

```
summary(logisticmodel)
```

Call:

```
glm(formula = outcome ~ var3, family = binomial(link = "logit"),
    data = X)
```

Coefficients:

```
              Estimate Std. Error z value Pr(>|z|)
(Intercept)  0.09357    0.40378   0.232   0.817
var3         -0.02958    0.09823  -0.301   0.763
```

(Dispersion parameter for binomial family taken to be 1)

```
Null deviance: 83.178  on 59  degrees of freedom
Residual deviance: 83.087  on 58  degrees of freedom
AIC: 87.087
```

Number of Fisher Scoring iterations: 3

```
# attach probabilities to raw data:
```

```
X$prob <- predict.glm(logisticmodel, X, type="response")
```

## 4 The apply-Family of Functions

### 4.1 Required Packages

No special packages required. All the apply-functions are included in the base package.

### 4.2 Create Sample Data

```
set.seed(123)
x <- matrix(rchisq(1000, df=3), ncol=4)
X <- as.data.frame(x)
X$V5 <- rep(c("A", "B", "C", "D", "E"), 50)
head(X)
```

|   | V1         | V2        | V3        | V4        | V5 |
|---|------------|-----------|-----------|-----------|----|
| 1 | 1.03611518 | 1.0692228 | 1.167547  | 3.1752303 | A  |
| 2 | 5.08870916 | 4.0810045 | 1.480796  | 0.7986091 | B  |
| 3 | 0.04818784 | 1.1842015 | 1.141494  | 0.9105581 | C  |
| 4 | 2.26693313 | 0.4574213 | 2.005387  | 0.7168796 | D  |
| 5 | 6.90085393 | 1.1796676 | 11.123718 | 3.8133264 | E  |
| 6 | 3.02805429 | 0.1332375 | 9.735566  | 0.2529623 | A  |

```
dim(X)
```

```
[1] 250  5
```

## 4.3 The apply Function

### Syntax

- **Description:** Returns a vector or array or list of values obtained by applying a function to margins of an array or matrix.
- **Usage:** `apply(X, MARGIN, FUN)`.
  - X is your “rectangular” data.
  - MARGIN 1 row-wise operation, 2: column-wise operation.
  - FUN - function to apply.

### Example

```
x1 <- apply(X[, -5], 1, sum)
length(x1)
```

```
[1] 250
```

```
# vector with 250 elements (i.e. number of rows in X)
```

## 4.4 The lapply Function

### Syntax

- **Description:** Apply a function over a list or vector and returns a list.
- **Usage:** `lapply(X, FUN)`.
  - X is your input data.
  - FUN - function to apply.

### Example

```
byV5 <- split(X, X$V5)
#byV5 is a list of data.frame objects.
x <- lapply(byV5, dim)

class(x)

[1] "list"

print(x)

$A
[1] 50  5

$B
[1] 50  5

$C
[1] 50  5

$D
[1] 50  5

$E
[1] 50  5
```

## 4.5 The sapply Function

### Syntax

- **Description:** Applies a function over a list or vector and returns a vector or an array.
- **Usage:** `sapply(X, FUN)`.
  - X is your input data.
  - FUN - function to apply.

### Example

```
byV5 <- split(X, X$V5)
x1 <- sapply(byV5, dim)
x1
```

```
      A  B  C  D  E
[1,] 50 50 50 50 50
[2,]  5  5  5  5  5
```

```
class(x1)
```

```
[1] "matrix" "array"
```

```
print(x1)
```

```
      A  B  C  D  E
[1,] 50 50 50 50 50
[2,]  5  5  5  5  5
```

## 4.6 The `tapply` Function

### Syntax

- **Description:** Applies a function to each cell of a ragged array, that is to each (non-empty) group of values given by a unique combination of the levels of certain factors.
- **Usage:** `tapply(X$var1, X$var2, FUN)`.
  - `X` is a `data.frame` (or similar structure, e.g. a `tibble`)
  - `X$var1` is the variable to analyze.
  - `X$var2` is the grouping variable.
  - `FUN` - function to apply, usually something related to statistics, e.g. the mean or the median.

### Examples

The average (arithmetic mean) for each group:

```
tapply(X$V1, X$V5, mean)
```

| A        | B        | C        | D        | E        |
|----------|----------|----------|----------|----------|
| 2.733125 | 2.949721 | 2.776092 | 2.456608 | 2.882719 |

The maximum value for each group:

```
tapply(X$V1, X$V5, max)
```

| A        | B        | C         | D         | E         |
|----------|----------|-----------|-----------|-----------|
| 8.815004 | 8.963654 | 10.252916 | 11.083424 | 14.252381 |

## 5 Advanced Visualisation with ggplot2 and cowplot

### 5.1 Required packages

- ggplot2 - for the plots
- dplyr - for data management
- cowplot - to combine multiple plots on one page

### 5.2 Create Sample Data

We use the following `data.frame`:

```
set.seed(123)
var1 <- round(c(rnorm(n=100, mean=30, sd=2), rnorm(n=100, mean=35, sd=2)), 3)
var2 <- var1 + rnorm(200, 0, 5)
set.seed(456)
var3 <- round(rchisq(n=200, df=3), 3)
var4 <- rep(c("A", "B"), 100)
var5 <- sample(c("X", "Y", "Z"), 200, replace=TRUE)
mydata <- data.frame(var1, var2, var3, var4, var5)
rm(var1, var2, var3, var4, var5)
head(mydata, 5)
```

|   | var1   | var2     | var3  | var4 | var5 |
|---|--------|----------|-------|------|------|
| 1 | 28.879 | 39.87305 | 0.215 | A    | Z    |
| 2 | 29.540 | 36.10206 | 4.638 | B    | X    |
| 3 | 33.117 | 31.79127 | 1.225 | A    | Z    |
| 4 | 30.141 | 32.85697 | 0.826 | B    | Y    |
| 5 | 30.259 | 28.18730 | 0.745 | A    | Y    |

### 5.3 Plots Using ggplot2 by Example

We create 12 representative statistical plots (some scatterplots, a piechart, a histogram, etc) and later combine them into three graphs using the function `plot_grid` in the `cowplot`-package. It is always possible to create graphics files with the following code:

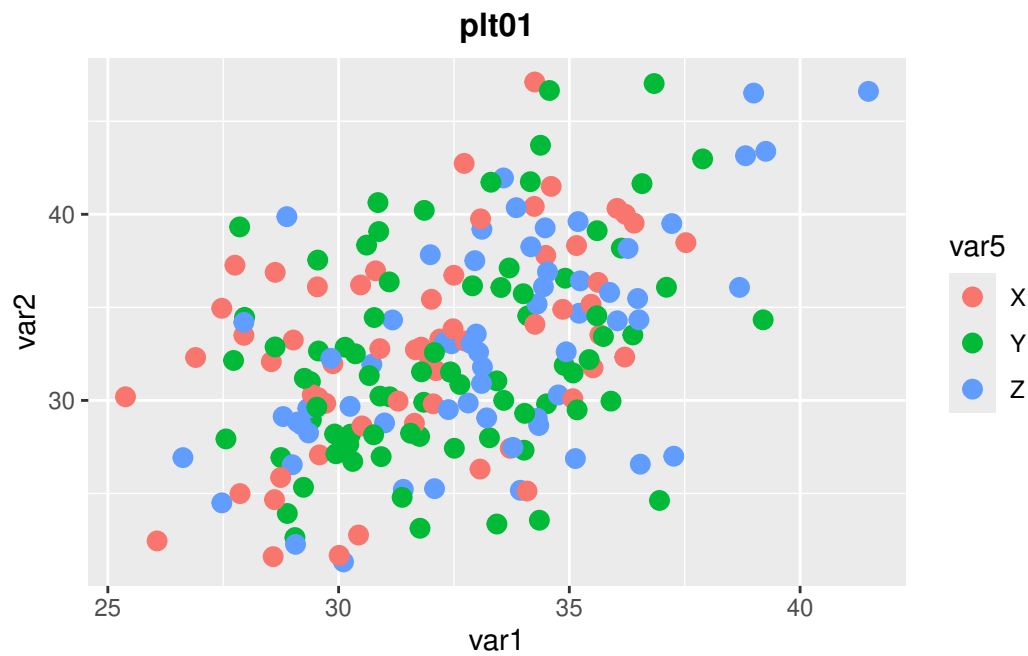
```
pdf("nameoffile.pdf") # or png(...), jpeg(...), ...
# your graphics commands here
dev.off()
```

## Plot 01 - A Simple Scatterplot

```
head(mydata, 3)
```

```
  var1    var2  var3 var4 var5
1 28.879 39.87305 0.215   A   Z
2 29.540 36.10206 4.638   B   X
3 33.117 31.79127 1.225   A   Z
```

```
library(ggplot2)
plt01 <- ggplot(mydata, aes(x=var1, y=var2, color=var5))
plt01 <- plt01 + geom_point(size=3)
plt01 <- plt01 + ggtitle("plt01")
plt01 <- plt01 + theme(plot.title=element_text(size=12, face="bold", hjust=0.5 ))
plt01
```

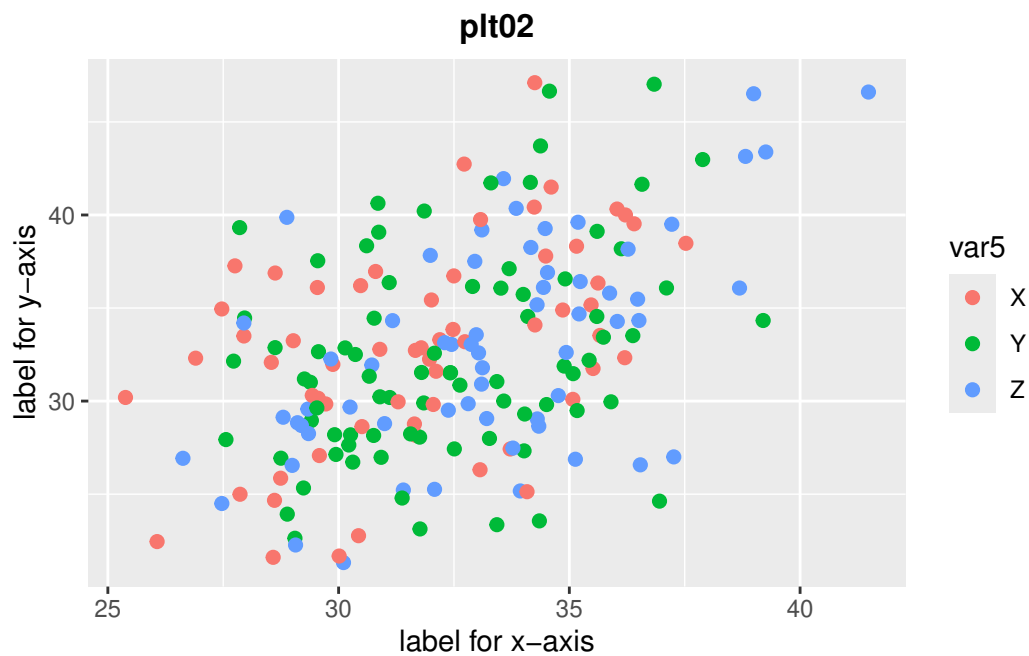


## Plot 02 - Adding More Text

```
head(mydata, 3)
```

```
  var1    var2  var3 var4 var5
1 28.879 39.87305 0.215   A   Z
2 29.540 36.10206 4.638   B   X
3 33.117 31.79127 1.225   A   Z
```

```
plt02 <- ggplot(mydata, aes(x=var1, y=var2, color=var5))
plt02 <- plt02 + geom_point(size=2)
plt02 <- plt02 + theme(plot.title=element_text(size=12, face="bold", hjust=0.5 ))
plt02 <- plt02 + xlab("label for x-axis")
plt02 <- plt02 + ylab("label for y-axis")
plt02 <- plt02 + ggtitle("plt02")
plt02
```



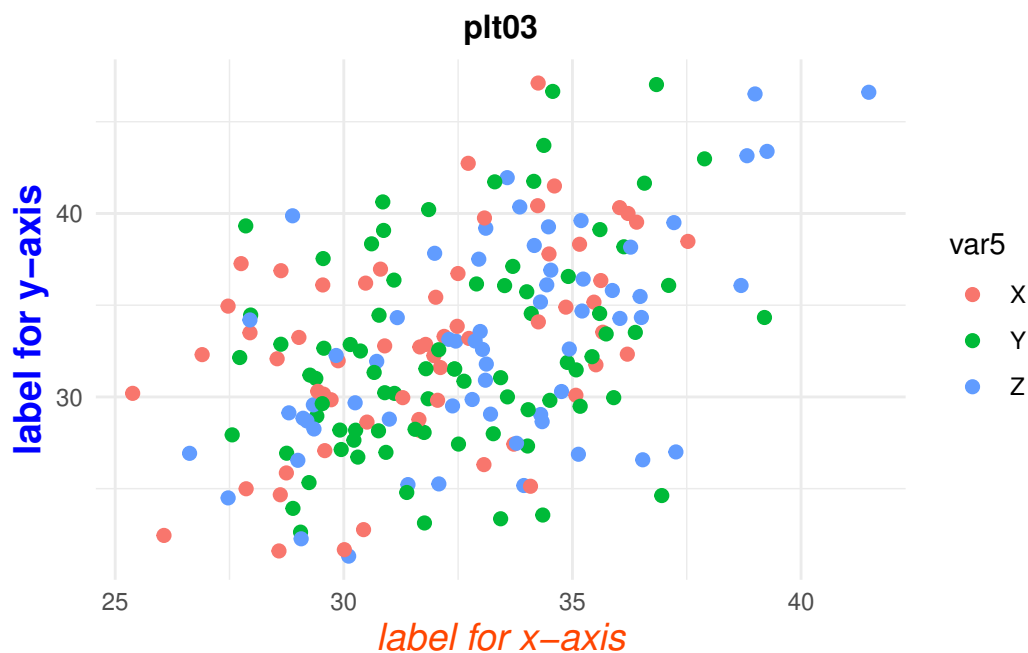
### Plot 03 - Formatting the Title and Annotations

```
head(mydata, 3)
```

```
      var1      var2  var3 var4 var5
1 28.879 39.87305 0.215   A   Z
2 29.540 36.10206 4.638   B   X
3 33.117 31.79127 1.225   A   Z
```

```
plt03 <- ggplot(mydata, aes(x=var1, y=var2, color=var5))
plt03 <- plt03 + geom_point(size=2)
plt03 <- plt03 + theme_minimal()
plt03 <- plt03 + theme(plot.title=element_text(size=12, face="bold", hjust=0.5 ))
plt03 <- plt03 + xlab("label for x-axis") + ylab("label for y-axis")
plt03 <- plt03 + theme(axis.title.x = element_text(color="orangered",
                                                    size=14, face="italic"))
plt03 <- plt03 + theme(axis.title.y = element_text(color="blue",
                                                    size=14, face="bold"))

plt03<- plt03 + ggtitle("plt03")
plt03
```



### **Explanation: Formatting Text**

With `element_text(...)` we change the appearance of texts, i.e. the title and axes annotations:

- `family` - font family.
- `face` - font face, e.g. “plain”, “italic” or “bold”.
- `colour` - (or `color`) - text color.
- `size` - text size in pts.
- `hjust` - horizontal justification (in `[0, 1]`).
- `vjust` - vertical justification (in `[0, 1]`).
- `lineheight` - line height (for multiline text only).

### **Explanation: Themes**

It is possible to use predefined templates for the plots (in the previous example I used `theme_minimal()`). Here are some of them:

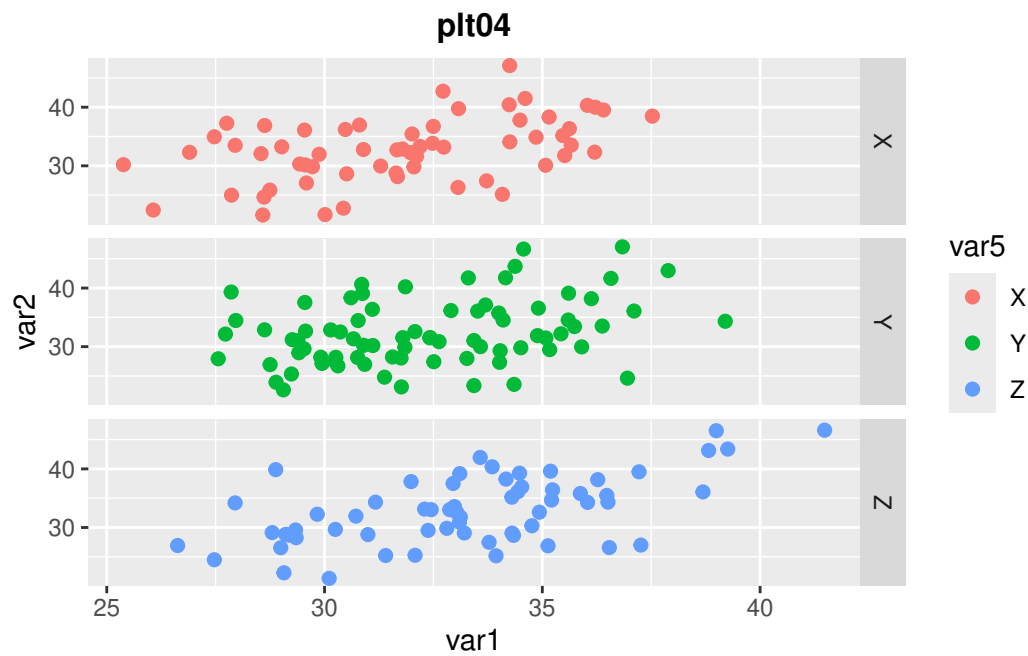
- `theme_grey()`
- `theme_bw()`
- `theme_light()`
- `theme_dark()`
- `theme_classic()`
- `theme_void()`

## Plot 04 - Using facet\_grid (Version 1)

```
head(mydata, 3)
```

```
  var1    var2  var3 var4 var5
1 28.879 39.87305 0.215   A   Z
2 29.540 36.10206 4.638   B   X
3 33.117 31.79127 1.225   A   Z
```

```
plt04 <- ggplot(mydata, aes(x=var1, y=var2, color=var5))
plt04 <- plt04 + geom_point(size=2)
plt04 <- plt04 + facet_grid(var5~.)
plt04 <- plt04 + theme(plot.title=element_text(size=12, face="bold", hjust=0.5 ))
plt04<- plt04 + ggtitle("plt04")
plt04
```

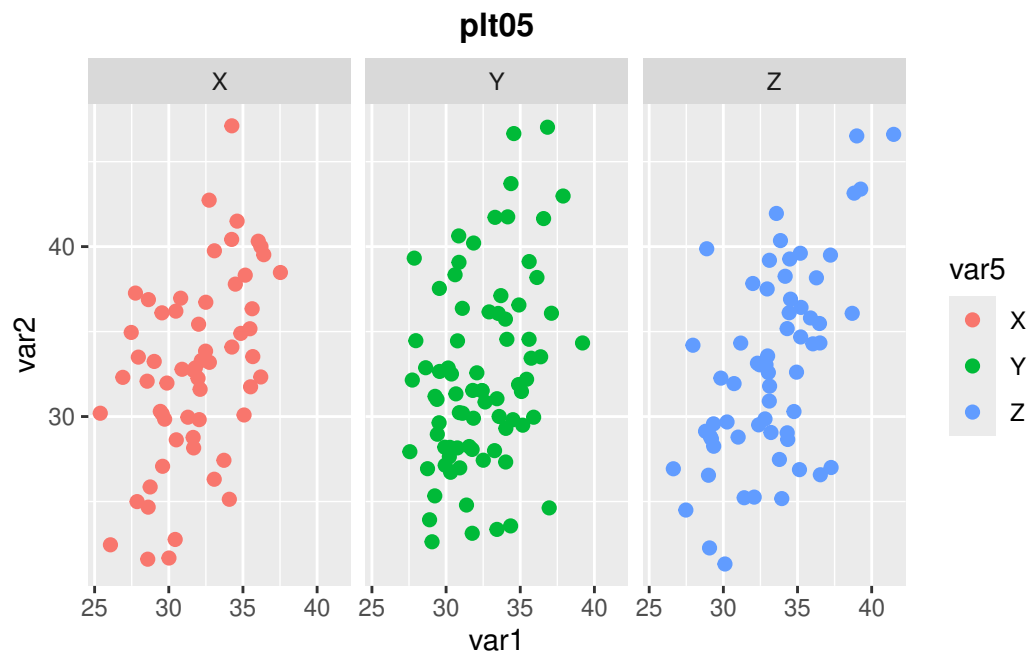


## Plot 05 - Using facet\_grid (Version 2)

```
head(mydata, 3)
```

```
  var1    var2 var3 var4 var5
1 28.879 39.87305 0.215   A   Z
2 29.540 36.10206 4.638   B   X
3 33.117 31.79127 1.225   A   Z
```

```
plt05 <- ggplot(mydata, aes(x=var1, y=var2, color=var5))
plt05 <- plt05 + geom_point(size=2)
plt05 <- plt05 + facet_grid(.~var5)
plt05 <- plt05 + theme(plot.title=element_text(size=12, face="bold", hjust=0.5 ))
plt05<- plt05 + ggtitle("plt05")
plt05
```

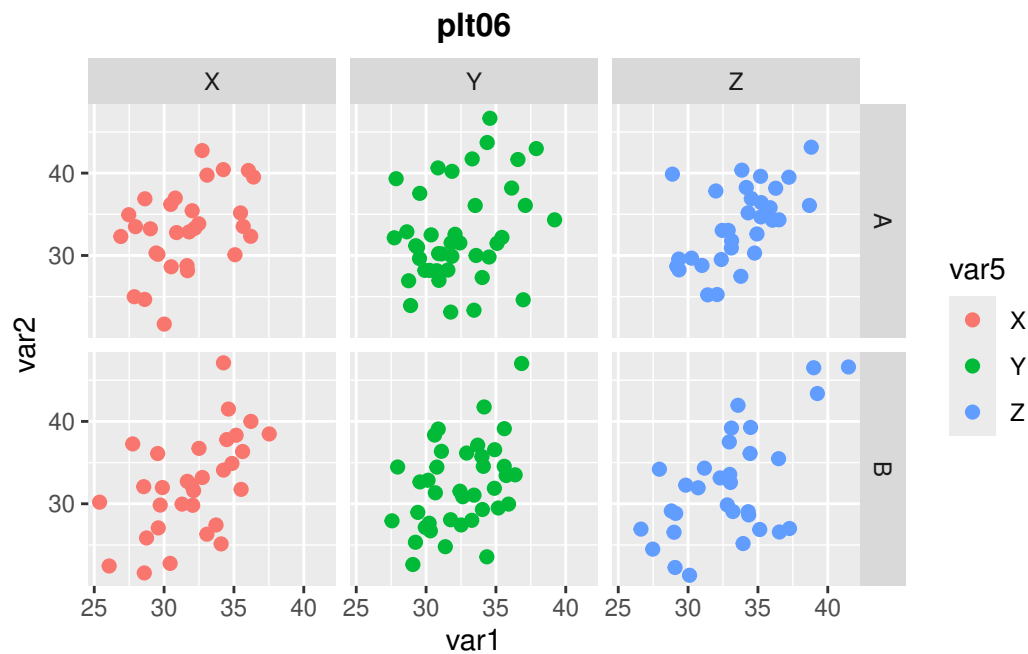


## Plot 06 - Using facet\_grid (Version 3)

```
head(mydata, 3)
```

```
  var1    var2  var3 var4 var5
1 28.879 39.87305 0.215   A   Z
2 29.540 36.10206 4.638   B   X
3 33.117 31.79127 1.225   A   Z
```

```
plt06 <- ggplot(mydata, aes(x=var1, y=var2, color=var5))
plt06 <- plt06 + geom_point(size=2)
plt06 <- plt06 + facet_grid(var4~var5)
plt06 <- plt06 + theme(plot.title=element_text(size=12, face="bold", hjust=0.5 ))
plt06<- plt06 + ggtitle("plt06")
plt06
```



## Plot 07 - A Barplot

```
head(mydata, 3)
```

```
   var1    var2  var3 var4 var5
1 28.879 39.87305 0.215   A   Z
2 29.540 36.10206 4.638   B   X
3 33.117 31.79127 1.225   A   Z
```

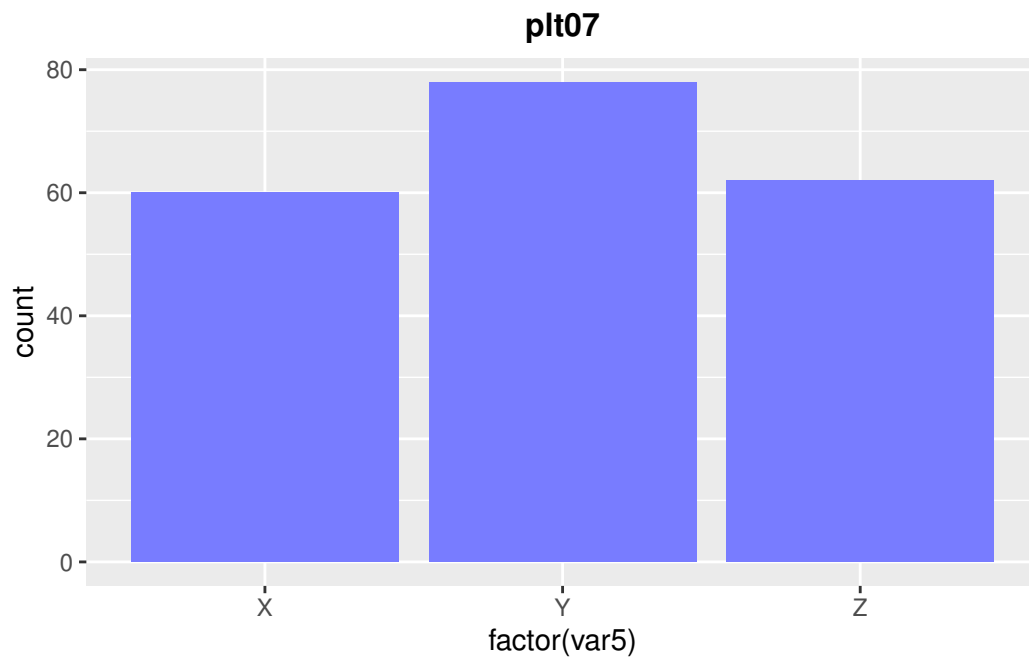
```
plt07 <- ggplot(mydata, aes(x=factor(var5)))
```

```
plt07 <- plt07 + geom_bar(stat="count", fill="#787cfc")
```

```
plt07 <- plt07 + theme(plot.title=element_text(size=12, face="bold", hjust=0.5 ))
```

```
plt07<- plt07 + ggtitle("plt07")
```

```
plt07
```

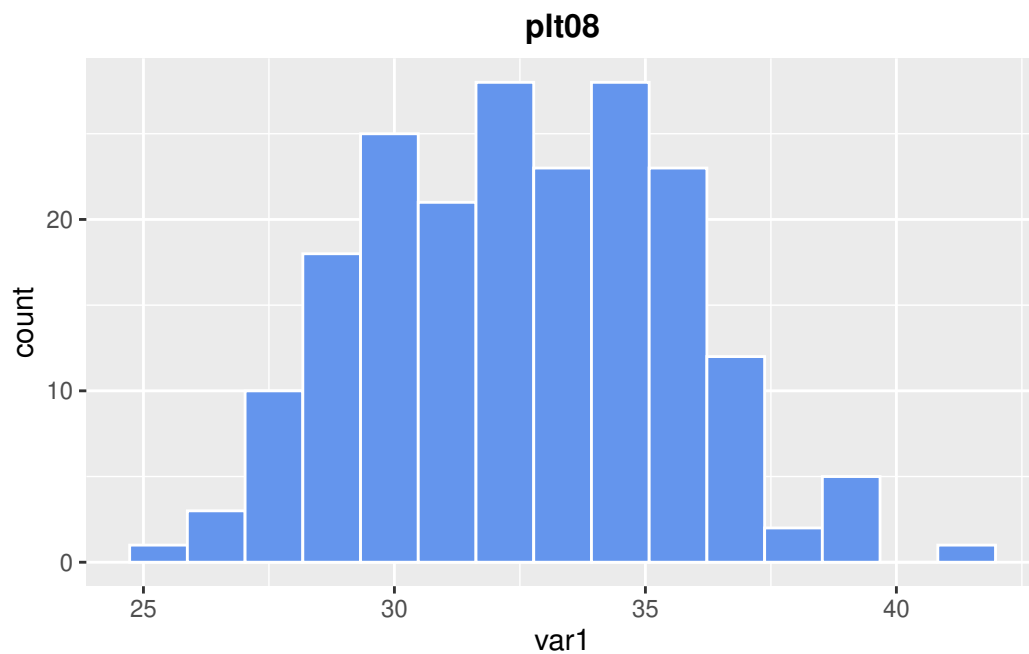


## Plot 08 - A Histogram

```
head(mydata, 3)
```

|   | var1   | var2     | var3  | var4 | var5 |
|---|--------|----------|-------|------|------|
| 1 | 28.879 | 39.87305 | 0.215 | A    | Z    |
| 2 | 29.540 | 36.10206 | 4.638 | B    | X    |
| 3 | 33.117 | 31.79127 | 1.225 | A    | Z    |

```
plt08 <- ggplot(mydata, aes(x=var1))  
plt08 <- plt08 + geom_histogram(color="white", fill="cornflowerblue", bins=15)  
plt08 <- plt08 + theme(plot.title=element_text(size=12, face="bold", hjust=0.5 ))  
plt08<- plt08 + ggtitle("plt08")  
plt08
```

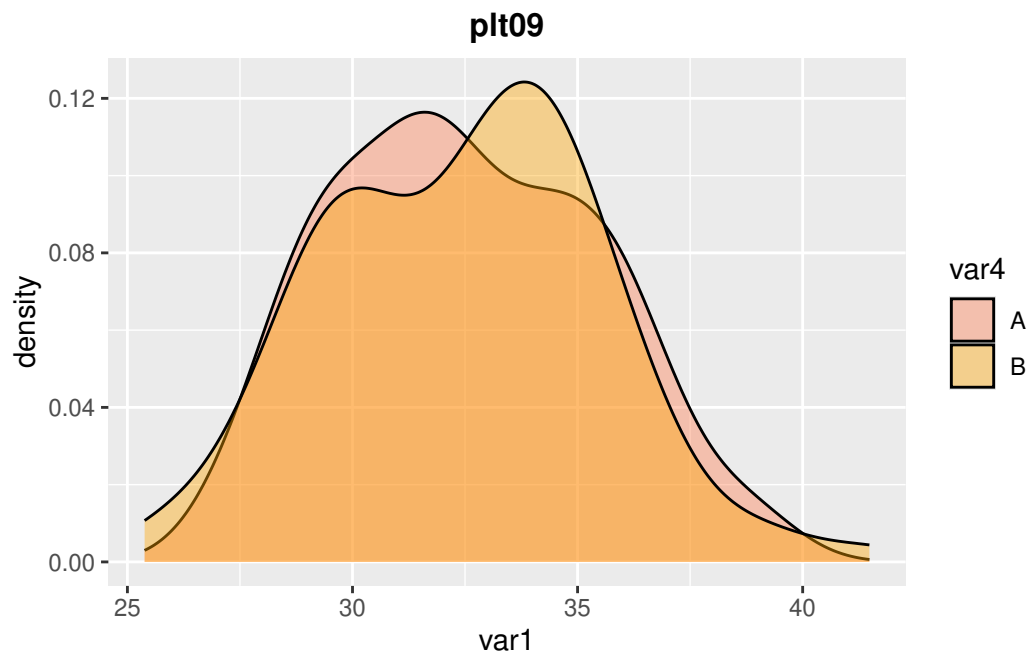


## Plot 09 - Densities

```
head(mydata, 3)
```

```
  var1    var2 var3 var4 var5
1 28.879 39.87305 0.215   A   Z
2 29.540 36.10206 4.638   B   X
3 33.117 31.79127 1.225   A   Z
```

```
plt09 <- ggplot(mydata, aes(x=var1, fill=var4))
plt09 <- plt09 + geom_density(alpha=0.4)
plt09 <- plt09 + scale_fill_manual(values=c("coral", "orange"))
plt09 <- plt09 + theme(plot.title=element_text(size=12, face="bold", hjust=0.5 ))
plt09<- plt09 + ggtitle("plt09")
plt09
```

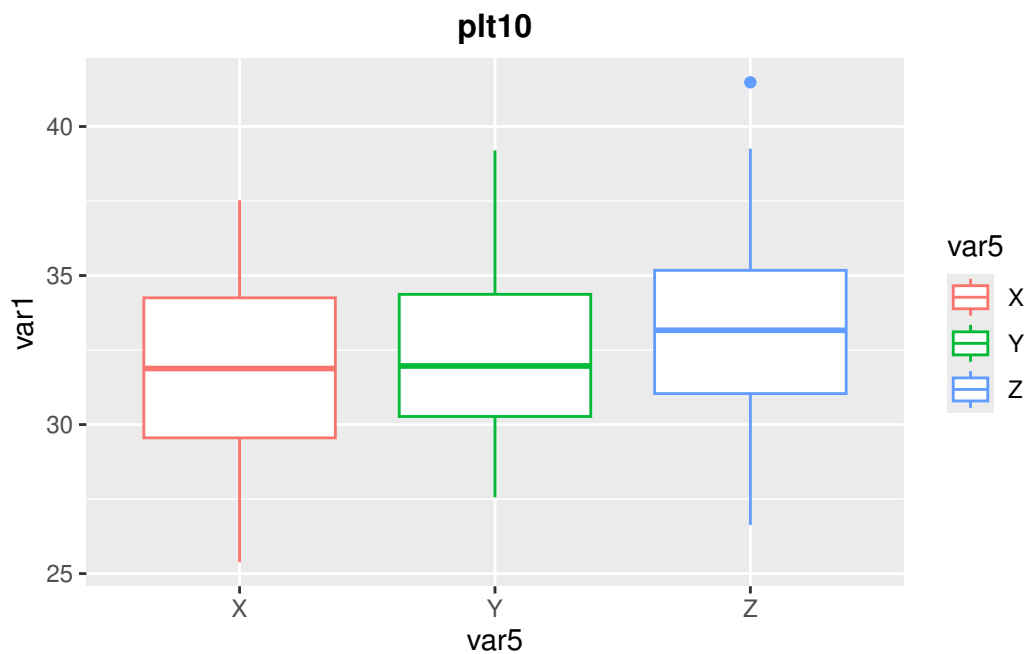


## Plot 10 - Boxplot by Group

```
head(mydata, 3)
```

```
  var1    var2 var3 var4 var5
1 28.879 39.87305 0.215   A   Z
2 29.540 36.10206 4.638   B   X
3 33.117 31.79127 1.225   A   Z
```

```
plt10 <- ggplot(mydata, aes(x=var5, y=var1, color=var5))
plt10 <- plt10 + geom_boxplot()
plt10 <- plt10 + theme(plot.title=element_text(size=12, face="bold", hjust=0.5 ))
plt10<- plt10 + ggtitle("plt10")
plt10
```



## Plot 11 - Another Barplot

```
head(mydata, 1)
```

```
      var1      var2  var3 var4 var5  
1 28.879 39.87305 0.215   A    Z
```

```
mytab = table(mydata$var5); mytab <- as.data.frame(mytab)
```

```
names(mytab) <- c("Value", "Frequency")
```

```
mytab
```

```
  Value Frequency  
1     X         60  
2     Y         78  
3     Z         62
```

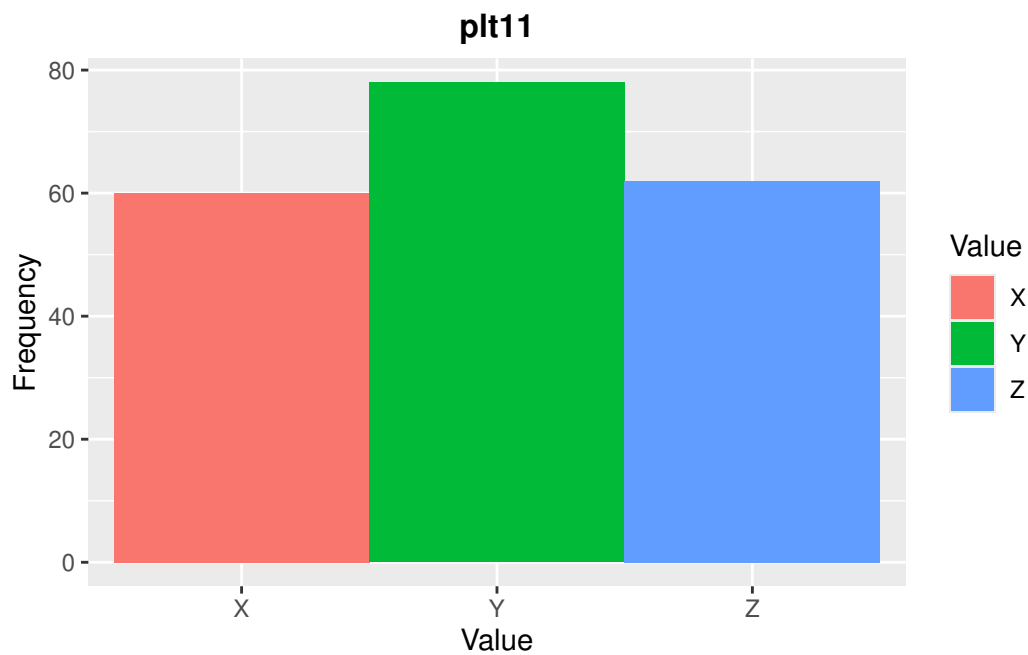
```
plt11 <- ggplot(mytab, aes( x=Value, y=Frequency, fill=Value))
```

```
plt11 <- plt11 + geom_bar(width=1, stat="identity")
```

```
plt11 <- plt11 + theme(plot.title=element_text(size=12, face="bold", hjust=0.5 ))
```

```
plt11<- plt11 + ggtitle("plt11")
```

```
plt11
```

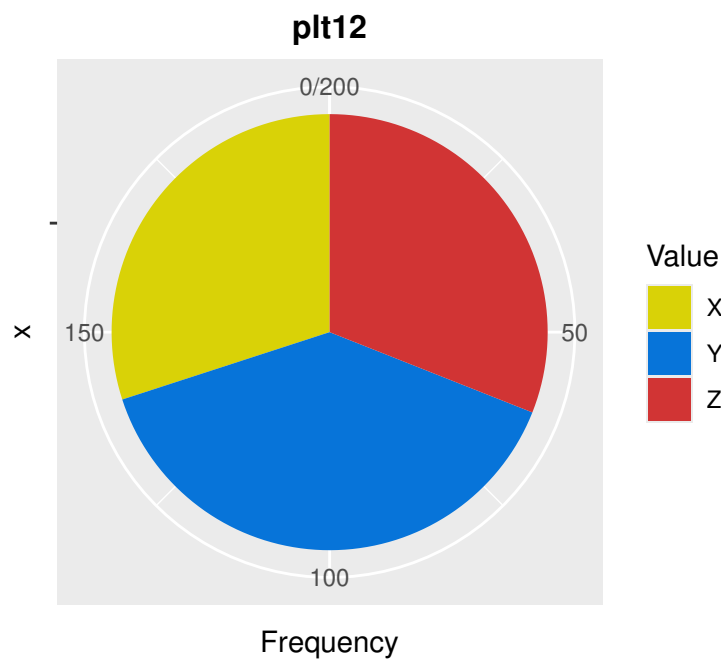


## Plot 12 - A Piechart

```
head(mydata, 3)
```

```
      var1      var2  var3 var4 var5
1 28.879 39.87305 0.215   A   Z
2 29.540 36.10206 4.638   B   X
3 33.117 31.79127 1.225   A   Z
```

```
mytab = table(mydata$var5)
mytab <- as.data.frame(mytab)
names(mytab) <- c("Value", "Frequency")
plt12 <- ggplot(mytab, aes(x="", y=Frequency, fill=Value))+
  geom_bar(width = 1, stat = "identity")
plt12 <- plt12 + coord_polar("y", start=0)
plt12 <- plt12 + scale_fill_manual(values=c("#d9d207", "#0774d9", "#d13434"))
plt12 <- plt12 + theme(plot.title=element_text(size=12, face="bold", hjust=0.5 ))
plt12<- plt12 + ggtitle("plt12")
plt12
```

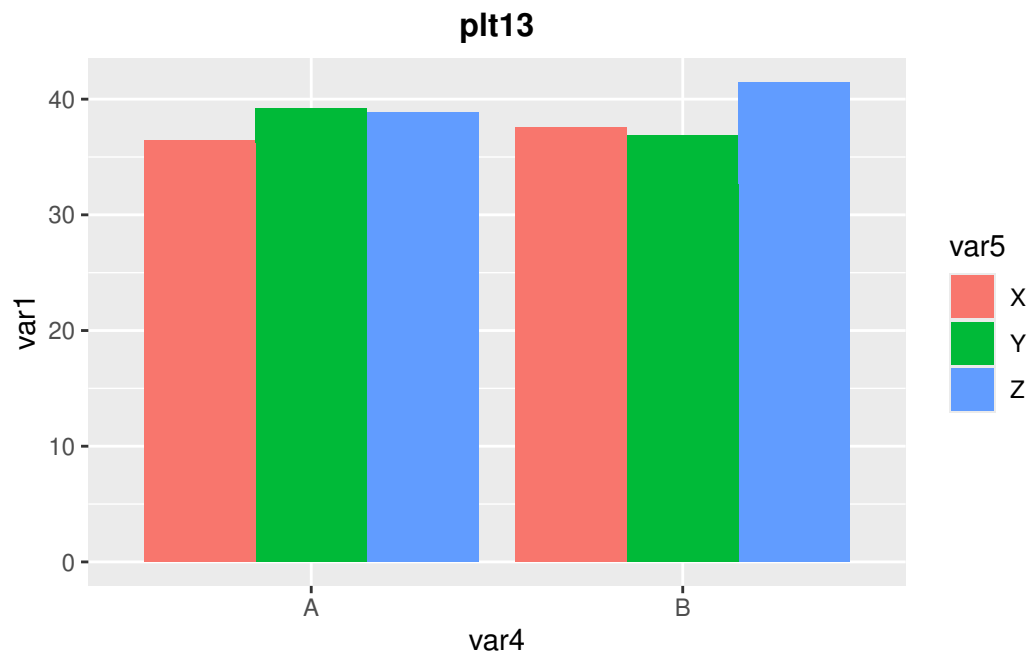


### Plot 13 - A Stacked Barplot

```
head(mydata, 3)
```

```
      var1      var2  var3 var4 var5
1 28.879 39.87305 0.215   A   Z
2 29.540 36.10206 4.638   B   X
3 33.117 31.79127 1.225   A   Z
```

```
plt13 <- ggplot(mydata, aes(fill=var5, y=var1, x=var4))
plt13 <- plt13 + geom_bar(position="dodge", stat="identity")
plt13 <- plt13 + ggtitle("plt13")
plt13 <- plt13 + theme(plot.title=element_text(size=12, face="bold", hjust=0.5 ))
plt13<- plt13 + ggtitle("plt13")
plt13
```

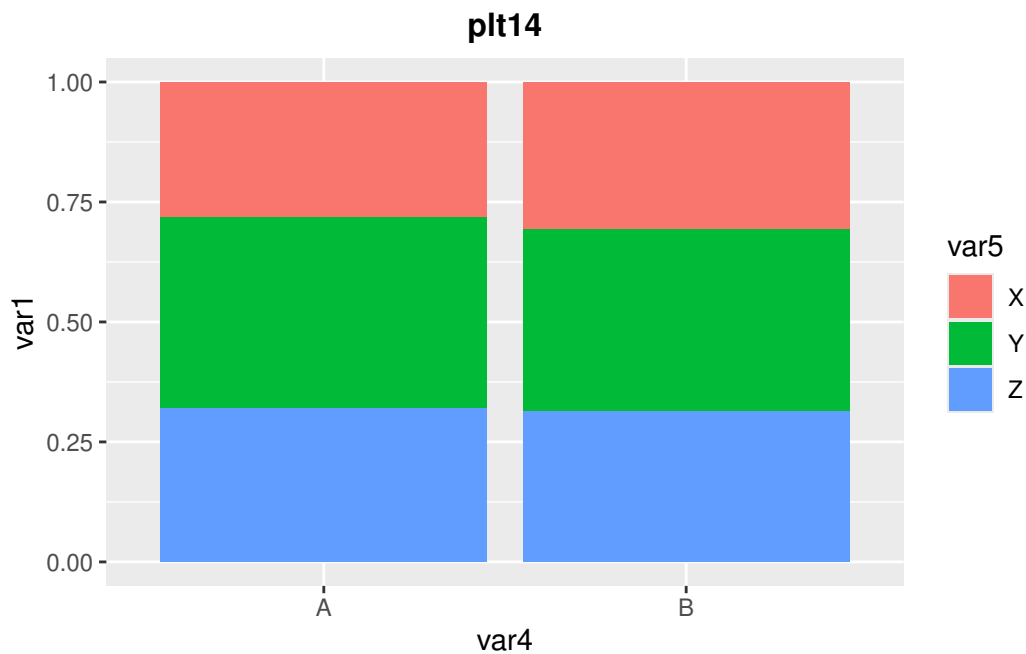


## Plot 14 - A Percent Stacked Barplot

```
head(mydata, 3)
```

```
      var1      var2 var3 var4 var5
1 28.879 39.87305 0.215   A   Z
2 29.540 36.10206 4.638   B   X
3 33.117 31.79127 1.225   A   Z
```

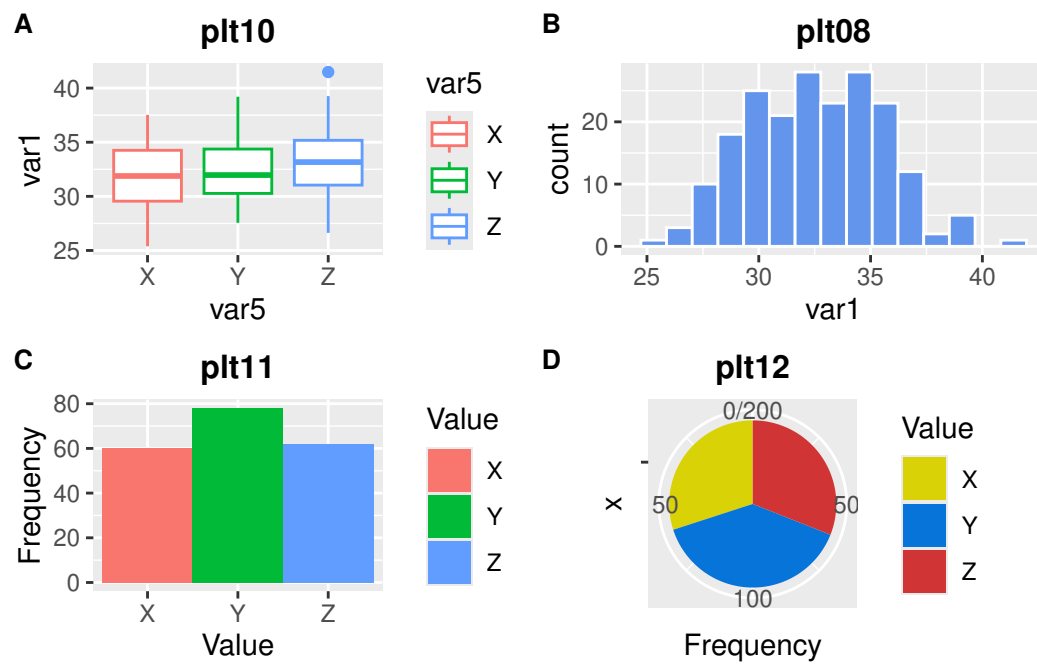
```
plt14 <- ggplot(mydata, aes(fill=var5, y=var1, x=var4))
plt14 <- plt14 + geom_bar(position="fill", stat="identity")
plt14 <- plt14 + ggtitle("plt14")
plt14 <- plt14 + theme(plot.title=element_text(size=12, face="bold", hjust=0.5 ))
plt14<- plt14 + ggtitle("plt14")
plt14
```



## 5.4 Combining the Plots

```
library(ggplot2)
library(cowplot)
```

```
plot_grid(plotlist = list(plt10, plt08, plt11, plt12),
labels = c("A", "B", "C", "D"), label_size = 10, nrow=2)
```



## 6 Loops and Simulation

Example: Calculate the test statistic for a contingency table of two variables. One variable has four possible values, the other one has two possible values. If we calculate the measure of discrepancy, say, 5000 times and save these numbers, the distribution of these measures should closely resemble a  $\chi^2$  distribution with  $(4 - 1) \times (2 - 1) = 3$  degrees of freedom.

Please install the `tidyverse` package first.

### Code

```
library(ggplot2)

simul <- function(n){
  values = rep(NA, n)

  for (i in 1:n)
  {
    x <- sample(c("A", "B", "C", "D"), 250, replace=TRUE)
    y <- sample(c("X", "Y"), 250, replace=TRUE)

    u <- table(x,y)

    result <- chisq.test(u)
    names(result)
    values[i] <- result$statistic
  }
  return(values)
}

n <- 5000
simresults <- simul(n)
# create a dataframe
mydata <- data.frame(simresults, theoretical = rchisq(n,3))
print(head(mydata,2))

  simresults theoretical
1   2.302526      4.935269
2   3.056930      2.433447
# randomly select data for Smirnov Test
mydata_part <- dplyr::slice_sample(mydata, prop=0.1)
# Smirnov Test
print(ks.test(mydata_part$simresults, mydata_part$theoretical))

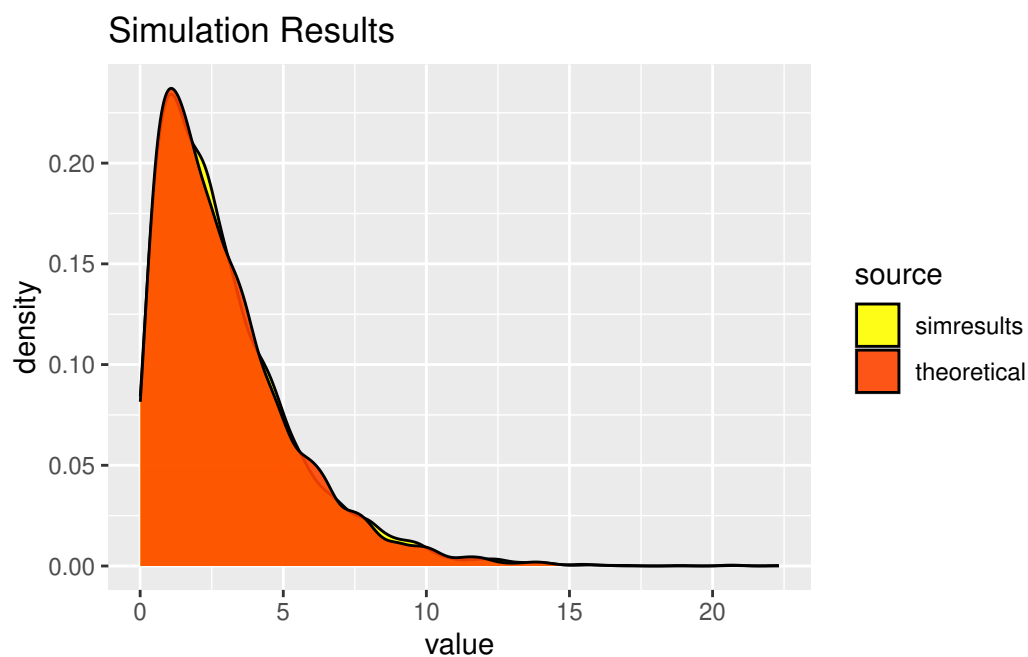
Asymptotic two-sample Kolmogorov-Smirnov test

data:  mydata_part$simresults and mydata_part$theoretical
D = 0.058, p-value = 0.3696
alternative hypothesis: two-sided
```

```
X <- tidyr::gather(mydata, key="source", value="value", simresults, theoretical)
print(head(X, 2))
```

```
      source    value
1 simresults 2.302526
2 simresults 3.056930

plt <- ggplot(X, aes(x=value, fill=source))
plt <- plt + geom_density(alpha=.9)
plt <- plt + scale_fill_manual(values=c("yellow", "orangered"))
plt <- plt + ggtitle("Simulation Results")
plt
```



As expected from theoretical considerations we get an empirical distribution that closely resembles a  $\chi^2$ -distribution with  $df = 3$ .

## 7 Functions for the matrix (and array) Object

### 7.1 Functions

| Function                                | Description                                          |
|-----------------------------------------|------------------------------------------------------|
| <code>matrix(data, nrow, byrow )</code> | Creates a matrix. For vectors use <code>c()</code> . |
| <code>array(data, dim)</code>           | Creates an array.                                    |
| <code>+, -, *, /</code>                 | Basic arithmetic operators.                          |
| <code>%*%</code>                        | Matrix multiplication.                               |
| <code>t(A)</code>                       | Transposes a matrix A.                               |
| <code>solve(A)</code>                   | Creates the inverse matrix $A^{-1}$ .                |
| <code>solve(A, b)</code>                | Solves the linear system $Ax = b$ .                  |
| <code>rank(A)</code>                    | Rank of a matrix A.                                  |
| <code>det(A)</code>                     | Determinant of a Matrix A.                           |
| <code>eigen(A)</code>                   | Calculates eigenvalues and eigenvectors.             |
| <code>qr(A)</code>                      | Computes the QR decomposition of a matrix.           |
| <code>crossprod(A, B)</code>            | Cross product.                                       |
| <code>tcrossprod(A, B)</code>           | Transposed cross product.                            |

### 7.2 Examples

```
A <- matrix(c(1,2,3,4), nrow=2, byrow=TRUE)
```

```
Ainv <- solve(A)
```

```
print(A)
```

```
      [,1] [,2]
[1,]     1     2
[2,]     3     4
```

```
print(Ainv)
```

```
      [,1] [,2]
[1,] -2.0  1.0
[2,]  1.5 -0.5
```

```
B <- A %*% Ainv
```

```
print(B)
```

```
      [,1]      [,2]
[1,]     1 1.110223e-16
[2,]     0 1.000000e+00
```

```
Z <- array(1:30, dim=c(2,5,3))
print(Z)
```

```
, , 1
```

|      | [,1] | [,2] | [,3] | [,4] | [,5] |
|------|------|------|------|------|------|
| [1,] | 1    | 3    | 5    | 7    | 9    |
| [2,] | 2    | 4    | 6    | 8    | 10   |

```
, , 2
```

|      | [,1] | [,2] | [,3] | [,4] | [,5] |
|------|------|------|------|------|------|
| [1,] | 11   | 13   | 15   | 17   | 19   |
| [2,] | 12   | 14   | 16   | 18   | 20   |

```
, , 3
```

|      | [,1] | [,2] | [,3] | [,4] | [,5] |
|------|------|------|------|------|------|
| [1,] | 21   | 23   | 25   | 27   | 29   |
| [2,] | 22   | 24   | 26   | 28   | 30   |

```
print(Z[1,2,3])
```

```
[1] 23
```

## 8 Recommended Reading

- Everitt, B., Hothorn, T. (2011). *An Introduction to Applied Multivariate Analysis with R*. Berlin: Springer.
- Kabacoff, R. (2011). *R in Action. Data Analysis and Graphics with R*. Shelter Island: Manning Publications.
- Ligges, U. (2008). *Programmieren mit R*. (3. Auflage). Berlin/Heidelberg: Springer.
- Sauer, S. (2019). *Moderne Datenanalyse mit R: Daten einlesen, aufbereiten, visualisieren, modellieren und kommunizieren*. Wiesbaden: Springer.
- Wickham, H., Golemund, G. (2016). *R for Data Science. Import, tidy, transform, visualize and model data*. Sebastopol: O'Reilly Media.
- Zelterman, D. (2015). *Applied Multivariate Statistics with R*. Berlin: Springer.